Teaching mathematics: The power of uncertainty

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Revealed: true extent of the crisis in maths

- Junior Cert grades have fallen
- State of education in maths

Our failure at maths

Failure rates in science and maths confirm system fears

US companies say Leaving Cert subject results 'disappointing'

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In the discussion document prepared for this conference, the role of the teacher in choosing appropriate mathematics problems and creating a problem-solving environment is addressed.

I argue that teachers also need to embrace their own ‘uncertainty’ as a crucial component of a problem-solving environment.
Preparing for the future

Barton (2013) describes how mathematical content is doubled every twenty-five years and that therefore it is impossible to prescribe core mathematics content for students. He goes on to suggest that, in order to prepare for the future, students needs to be confident in mathematics and to grasp its power and pleasure.
Views of mathematics

An objective, immutable body of facts to be transmitted from generation to generation

or

A cultural activity made up of different social practices

(e.g., Ernest, 2004)

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When is this statement true?

\[ 3 + 2 = 5 \]

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Even within arithmetical structures where $1 + 1$ does indeed always come out to 2, that is not the end of the matter. Quite the opposite, that is when things start getting interesting. We still need to think about what is being added to what, by whom and why. We need to think about what meanings people give to the adding they are doing and how it changes them and the world. We also need to engage with the ways that the idea of the possibility of certain knowledge acts to steer us away from these and other difficult questions; thus we need to confront the ideological functions of certainty.

(Mendick, 2006, p.158)
What are the implications of having five pieces of fruit? Are these implications the same in all contexts?

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Clean or messy?

The front and back of mathematics aren’t physical locations like dining room and kitchen. They’re its public and private aspects. The front is open to outsiders; the back is restricted to insiders. The front is mathematics in finished form – lectures, textbooks, journals. The back is mathematics among working mathematicians, told in offices or at café tables … Front mathematics is formal, precise, ordered and abstract. It’s broken into definitions, theorems and remarks… Mathematics in the back is fragmentary, informal, intuitive, tentative. We try this or that. We say ‘maybe’ or ‘looks like’. (Hersh, 1997, p.36)
Should mathematics be presented to students as ...
Clean and well-ordered...
... or busy but messy?
Boaler and Greeno (2000) suggest that an emphasis on ‘procedures-based’ mathematics positions learners of the subject as ‘received knowers’ (whereby they learn mathematics by carefully following teachers and textbook demonstrations) and eliminates creative, divergent thinkers from the subject. They also maintain that it has a critical effect on the number of students who want to study mathematics at advanced levels.
Research

Research Question

• What are the conditions under which mathematical insight is constructed by pupils in whole-class conversation?

Data (from 3 different schools)

• Audio tape recordings (whole class and grouped/paired work; follow-up interviews)
• ‘Ways of thinking’ sheets
• Digital photographs
• Pupils’ reflective diaries
• Field notes
• Researcher’s diary

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Background

- 5th class in a school of middle SES located in a provincial town in Ireland

- 30 pupils – 17 girls and 13 boys

- One lesson taught to the class was based on the ‘Grasshopper’ problem.
Grasshopper Problem

Line (‘mat’) drawn on blackboard, the initial point of which was marked ‘0’ and the endpoint ‘1m’. Pupils were asked to name the landing-points if the grasshopper jumped half-way across, then half-way across the remaining part and so on.
Zeno’s paradox of motion

What is suggested in this paradox is that one can never reach the end of a racetrack for in order to do so, one has first to reach the half-way mark, than the half-way mark of the remaining half, then the halfway mark of the next part and so on. Zeno contended that one can never reach the end of the racetrack, at least not in finite time. (Rucker, 1982)

There is conflict between ‘practical’ (real-life) and ‘theoretical’ (pure mathematical) considerations. (Stern and Mavarech, 1996)
Some pupils had difficulty naming the landing-point after ‘7/8’ - this was not surprising as ‘sixteenths’ are not part of the 5th class mathematics programme.

In the following excerpt the solutions to the problem offered by Kate and Jack are described
TD: What do you think is going to happen next?
Chn: It’s going to half it//half it//half
Jack: It’s half of seven eighths (*whisper*)

... 
Jack: He’s on fifteen sixteenths.
//Ch (Kate): Seven and a half eighths
Chn: No
TD: He’s on fifteen sixteenths, seven and a half eighths or fifteen…
//Ch talking …So you think he’s on fifteen sixteenths. Where are you getting fifteen sixteenths from?
Jack: Cos I think, I think em … I think a half an eighth is sixteen …

TD: Right.

Jack: and eh …

//Ch: I know

Jack: when …

//Chn: Seven and a half!

Jack: and then another sixteenth … if she went another sixteenth (other children talking in background), she’d be there but she didn’t go another sixteenth, so she went fifteen sixteenths.

Ch: (In background) Three quarters …
137 TD: Fifteen sixteenths … and who said seven and a half eighths, who said that?

…

143 Kate: It was seven eighths and if he went eight eighths, he would be at the end, so if you go half of it, then it’s seven and a half.

144 Mr. Allen: Good girl!

145 TD: Seven and a half eighths and do you think …

…

159 TD: … So what do we call … will we call it seven and a half eighths or fifteen sixteenths?

160 Chn: Seven and a half //Fifteenth sixteenths//Seven and a half is easier to manage// No, it’s not//Cos you are going two, four, eight//Seven and a half is easier (arguing)
Both solutions were written on the blackboard

An appropriate response?
Contingency scenario

159 TD: The jumps … oh, I know what you mean. So what do we call … will we call it seven and a half eighths or fifteen sixteenths?

160 Chn: Seven and a half // Fifteenth sixteenths // Seven and a half is easier to manage // No, it’s not // Cos you are going two, four, eight // Seven and a half is easier (arguing)

161 TD: I think … you could call it seven and a half eighths but we normally these things … normally they are brought up to full numbers. But seven and a half eighths would be ok but … normally it’s brought up to something like fifteen sixteenths. (I write both on blackboard)

See Rowland, Huckstep and Thwaites (2005); Corcoran (2012)
A dance of agency

“[T]eaching is a practice that takes place at the intersection of hundreds of variables that play out differently in every moment”. (Boaler, 2003, p.12)

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Possible follow-up...

- What if we named fractions using Kate’s way?
- Are Kate’s and Jack’s ways the same or different?
- Are there other ways we might name the landing point?

...
‘Getting comfortable with being uncomfortable’*

[Problem-solving environments] require new – sometimes uncomfortable - roles for both teachers and students. For students to ‘construct knowledge’ as advocated by these curricular programs, teachers are expected to model problem solving, explore real world contexts, value multiple solution strategies, and give students time to create, discuss, hypothesize, and investigate … Understanding how teachers respond to uncertainties that arise in situations such as these is important if innovative curriculum materials, and the broader contemporary mathematics education reform movement itself are to be successfully implemented. (Frykholm, 2004, p.126 – 127)

*Mau (2007, p.379)
Implications for teaching
Discussions at school and societal levels about what it means to ‘do mathematics’
Developing rich mathematical tasks

What two numbers add together to give 5?

3 + ...

3 + 2

2 + 3

-5 + 10?

4 ½ + ½

5 + 0

When/why?

How many ways?

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Interrogating texts and tests

1. Find the value of
   (c) 38 − 2  (b) 24 − 6  (c) 70 − 9
   (d) 50 − 30  (e) 70 − 20  (f) 90 − 50
   (g) 51 − 30  (h) 78 − 20  (i) 95 − 50

2. Find the value of
   (a) 230 − 7  (b) 206 − 9  (c) 411 − 8
   (d) 780 − 60  (e) 450 − 70  (f) 540 − 80
   (g) 500 − 200  (h) 700 − 400  (i) 900 − 300
   (j) 542 − 200  (k) 753 − 400  (l) 908 − 300

3. Subtract 23 from 54.

   54 \[\begin{array}{c}
   \downarrow \\
   34 \downarrow \\
   31
   \end{array}\]

   Subtract 20 from 54 first.

   54 − 20 \[\begin{array}{c}
   \downarrow \\
   34 \downarrow \\
   31
   \end{array}\]

   54 − 23 =

4. Find the value of
   (c) 58 − 17  (b) 87 − 16  (c) 42 − 12
   (d) 32 − 25  (e) 65 − 48  (f) 75 − 56

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Shifting ‘power’

• Asking pupils questions such as the following:
  Which of these (exercises in text) do you find easy/difficult/
  interesting/useful etc. (?)

• Giving pupils space and time for ‘possibility thinking’ (Craft,
  2000)

• Developing a ‘conjecturing atmosphere’ through probing and
  revoicing (e.g., broadcasting to class) individual pupils’
  contributions (Dooley, 2010)
Building mathematical knowledge for teaching

Corcoran (2012) suggests that reflection on contingency scenarios offers rich opportunities for building mathematical knowledge – particularly in a collaborative setting. Implications of this are (i) acknowledging that teacher discomfort is a component of a problem-solving environment and (ii) making such moments public and presenting them to others for reflection and discussion.
Teacher Efficacy

- Developing rich tasks
- Predicting (possible) student reasoning
- Generating and directing discussion
- Transformative moments
- …

(Smith III, 1996)
...and the grasshopper!

270 Maeve: It’s just like going to Cork, he will get there.
References


