Irish National Teachers' Organisation
Cumann Múinteoirí Éireann

Numeracy in the Primary School

A nation's greatness depends upon the education of its people

Numeracy in the Primary School

Discussion Document and Proceedings of the Consultative Conference on Education 2013

Numeracy in the Primary School

Discussion Document and
Proceedings of the Consultative
Conference on Education 2013

Irish National Teachers' Organisation
Vere Foster House
35 Parnell Square
Dublin 1

Telephone: 01 804 7700
Fax: 01 872 2462
Email: info@into.ie
Web: http://www.into.ie

General Secretary: Sheila Nunan

Cumann Múinteoirí Éireann
Áras Vere Foster
35 Cearnóg Parnell
Baile Átha Cliath 1

Guthán: 01 804 7700
Fax: 01 872 2462
Ríomhphost: info@into.ie
Gréasán: http://into.ie

Ard Rúnaí: Sheila Nunan

into
Irish National Teachers' Organisation
Cumann Múinteoirí Éireann

Table of Contents

Acknowledgements

Guest Presenters	Thérèse Dooley, St Patrick's College John Coolahan, NUI Maynooth Seán Delaney, Marino Institute of Education
Education Committee	Dympna Mulkerrins, Cathaoirleach Ger Stack, Leas-Chathaoirleach Geraldine McGowan Tommy McGlone Tina McDaid Pat Collins Karen Devine Siobhán Lynskey Breda Fay Anne English Martin Lynch Milo Walsh Aidan Gaughran Gerry O'Sullivan Áine Dillon Alice O'Connell
INTO Education Team	Claire Garvey Ann McConnell
Compilation and Editing	Deirbhile Nic Craith

Foreword

In Ireland, while our tradition in mathematics may not be as prominent as our tradition in literacy, the contribution of mathematicians such George Boole and William Rowan Hamilton to the development of mathematics in Ireland should be celebrated. There is no doubt about the importance of mathematics for every pupil in our education system. Since the publication in 2011 of the national strategy to improve literacy and numeracy in our schools, mathematics or numeracy in our schools has gained increasing attention. At the same time, there has also been an increase in the attention given to Ireland's participation in national and international assessments in mathematics. It is a policy objective to improve Ireland's performance in such assessments.

As we face into a process of reviewing the curriculum for mathematics at primary level, it is timely to consider how mathematics is both taught and learnt. The consultative conference on education in 2013 focused on this topic, bringing together guest presenters and experts in mathematics teaching in Ireland at primary level, to share their thoughts with teachers. Participation in the annual INTO consultative conference on education is a rich professional development experience for participating teachers and the 2013 conference presented an ideal opportunity for teachers to discuss and reflect on the issues in depth and to share their own experiences of mathematics teaching.

This publication includes a discussion document on mathematics in the primary school prepared by the INTO Education Committee. It describes mathematical literacy, drawing on a number of sources and outlines the main dimensions of the current primary school curriculum, in terms of content and assessment. The recommendations in the national strategy to improve literacy and numeracy pertaining to mathematics are summarised. The document also includes a description of the mathematics curriculum in Northern Ireland. Challenging aspects of mathematics teaching are outlined, including the language of maths, which needs discrete teaching, problem-solving and differentiation in mathematics teaching to ensure that all children can access mathematics. In addition, it cannot be ignored that there are many influences on children's learning in mathematics. The document then outlines some of the findings from the INTO survey on numeracy in primary schools which was carried out in 2011.

Part Two of the publication presents the proceedings of the consultative conference on education. We are grateful to our guest presenters, Thérèse Dooley, of St Patrick's College and Sean Delaney, Marino Institute of Education, whose high-quality presentations stimulated much of the discussion at the conference. The report of the discussion groups reflects the views of teachers on many aspects of mathematics discussed at the conference.

Our thanks are also due to members of the Education Committee who prepared the discussion document, presented at the conference and acted as facilitators and rapporteurs in the discussion groups. The contribution of our two interns Hazel O'Connor and Andrew Bowen is also acknowledged. The work of the Education Committee is coordinated by the education section of the organisation under the direction of Deirbhile Nic Craith, Director of Education and Research, Claire Garvey and Ann McConnell.

This publication forms part of the ongoing contribution by the INTO, on behalf of teachers, to the continuum of curriculum development in Ireland.

Sheila Nunan
General Secretary

Part 1

Numeracy in the Primary School

Discussion Document

1

Introduction

> The discipline of mathematics has been around for many generations, but the expectation that ordinary citizens be quantitatively literate has arisen in relatively recent times (de Lange[1]).

In preparing this discussion document, the terms 'numeracy' and 'mathematics' were both used, but their precise meanings are not the same. The Primary School Curriculum includes numeracy, and while not giving an explicit definition, the curriculum emphasises the importance of basic mathematics.

According to O'Donoghue (2002), there is no universally accepted definition of numeracy, even though the term is often used in discourse and policy. Cockcroft (1982) identified the Crowther Report (1959, UK) as the source of the term numeracy. The Crowther Report viewed numeracy as an important personal attribute, similar to literacy. Many definitions of literacy are inclusive of numeracy (O'Donoghue, 2002). Cockcroft, in his report on the teaching of mathematics in schools in the UK, *Mathematics Counts* (1982), defined numeracy as follows:

> We would wish the word numerate to imply the possession of two attributes. The first of these is an 'at homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical demands of everyday life. The second is ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. (Cockcroft, 1982, par. 39)

In broad terms mathematics is understood as including numeracy, which is often interpreted as the ability to use mathematics in everyday life. The use of mathematics in everyday activities include: computation and problem-solving, handling money, reading timetables and interpreting graphs.[2] According to the New South Wales Department of Education and Training, a numerate person understands the use of mathematics for communication, is at ease 'with all those aspects of mathematics that enable a person to cope with the practical demands of everyday life' and has 'the ability to understand information presented in mathematical terms'.[3] Individuals interpret the world by drawing on their understandings of number, measurement, probability, data and spatial sense combined with critical mathematical thinking[4]. The concepts of mathematical

[1] http://www.criced.tsukuba.ac.jp/math/sympo_2006/lange.pdf
[2] www.nationalnumeracy.org.uk/numeracy-vs-maths/index.html
[3] www.curriculumsupport.education.nsw.gov.au/primary/mathematics/numeracy/what/index.htm
[4] www.curriculumsupport.education.nsw.gov.au/primary/mathematics/numeracy/what/index.htm

literacy, numeracy and quantitative literacy are inter-related and their precise meanings continue to be debated.

What is mathematical literacy?

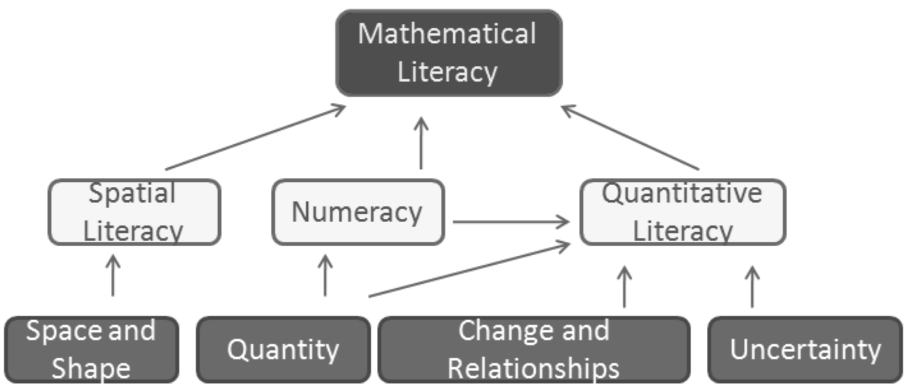
Literacy and numeracy have remained bracketed together in the public mind, and numeracy issues are often subsumed under literacy. Literacy still influences all major discussions on numeracy, and numeracy has been seen as 'an analogue of literacy'. Competence in Mathematical Literacy is also, of necessity, irrevocably bound up with competence in language. In essence, mathematics literacy is emerging as the knowledge to know and apply basic mathematics in our everyday lives (de Lange[5]). An understanding of numeracy as the ability to use mathematics is similar to the interpretation of mathematical literacy, used by the OECD in the international PISA assessment:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2006, PISA, p. 72)

The OECD's definition as used in PISA, suggests that pupils in schools need to develop skills for problem-solving, information-processing, decision-making and interpreting data, in addition to understanding numbers and mathematics.

De Lange's tree structure of mathematical literacy provides a useful approach to understanding mathematical literacy, which is more than ability to use mathematical procedures in everyday life. According to de Lange mathematical literacy is mathematical knowledge, methods and processes applied in various contexts and includes numeracy, quantitative literacy and spatial literacy. See figure 1 below.

Figure 1: **De Lange's tree structure of mathematical literacy**[6].



Spatial literacy is about empowering people to understand the three-dimensional world in which they live and move, requiring an understanding of properties of objects, the relative positions of objects, and practices such as navigation. Numeracy is understood as the ability to handle numbers and data in a variety of real life contexts such as

[5] http://www.criced.tsukuba.ac.jp/math/sympo_2006/lange.pdf
[6] http://math4teaching.com/2010/03/12/what-is-mathematical-literacy/

problem-solving and estimations. Quantitative literacy expands numeracy to include use of mathematics in dealing with change, quantitative relationships and uncertainties.

The need to be literate about mathematics and its applications is an important aspect of a mathematics curriculum in primary schools if individuals are to develop an understanding of core mathematical concepts and processes and their use in everyday life. Learning mathematics therefore should not be isolated from student experiences (de Lange). Students need to be engaged in problem-solving and investigation activities, using mathematics to reason and communicate.

To teach mathematical literacy, curriculum and instruction should, therefore, include these 3 Rs:

Relevant mathematical concepts, principles and procedures
Real-life context which can be investigated and modelled mathematically
Rich mathematical tasks that fosters conceptual understanding and development of skills and habits of mind[7]

The Organisation of Economic Co-operation and Development (OECD, 2002) claims that teaching students to 'mathematize' should be a primary goal of mathematics education. Mathematizing involves five elements:

1. Starting with a problem whose roots are situated in reality
2. Organizing the information and data according to mathematical concepts
3. Transforming a real-world, concrete application to an abstract problem whose roots are situated in mathematics
4. Solving the mathematical problem
5. Reflecting back from the mathematical solution to the real-world situation to determine whether the answer makes sense.

According to Steen (1997), mathematicians offer various definitions of quantitative and mathematical literacy. For example, quantitative literacy involves reasoning with numbers (Jim Lewis); reading, interpreting and making simple applications (Carole Lacampagne); understanding operations on rational numbers (Jack Price); constructing and recognizing a sound argument (Keith Devlin); and understanding variability and how to quantify it (Gail Burrill). In contrast, mathematical literacy offers a big-picture view of how to work with numbers, relationships, and patterns (Jim Lewis); higher order thinking, including all the goals of the NCTM Standards (Jack Price); mostly qualitative issues, not quantitative ones (Keith Devlin); the language of algebra as well as geometric and spatial experience (Zal Usiskin). *(Why Numbers Count: Quantitative Literacy for Tomorrow's America, Steen, 1997)*[8]

Outlining its new framework for assessment, the OECD (1999) listed types of texts, widely used in real life contexts, which entailed a grasp of quantitative literacy necessary for functional literacy. These included various types of forms and information sheets, in short, texts that require a grasp of numbers and mathematical concepts. For example:

Forms: tax forms, immigration forms, visa forms, application forms, questionnaires.

[7] http://math4teaching.com/2010/03/12/what-is-mathematical-literacy/
[8] http://www.stolaf.edu/other/extend/Numeracy/defns.html

Information sheets: timetables, price lists, catalogues, programs, vouchers, tickets, invoices, certificates, diplomas, contracts.
Charts and graphs: iconic representations of data, diagrams, tables and matrices, lists.

The concept of what it means to be mathematically literate is becoming increasingly important. Students need to develop confidence with mathematics, know how to interpret data, to think logically, to reason carefully, to understand arguments, to evaluate decisions, to understand the meaning of numbers, and to be able to estimate – all aspects of mathematics that one is likely to encounter at home or at work. (The Case for Quantitative Literacy[9])

Similarly, Jan de Lange[10] outlines the competencies which mathematical literacy requires:

- Mathematical thinking and reasoning.
- Mathematical argumentation.
- Mathematical communication.
- Modelling.
- Problem posing and solving.
- Representation.
- Symbols.
- Tools and technology.

According to Ojose (2011), a mathematically literate person should be able to estimate, interpret data, solve day-to-day problems, reason in numerical, graphical, and geometric situations, and communicate using mathematics. Mathematics literacy is necessary both at work and in daily life and is as important as proficiency in reading and writing. The skills and competences needed continue to be debated. However, there is an emerging view that having the ability to use the knowledge of basic maths to solve real-life problems is what is most necessary. Ojose describes the knowledge and competences required as follows:

> In arithmetic, everyone should be able to perform the basic operations of addition, subtraction, multiplication, and division in whole numbers, fractions and decimals. They should also know concepts such as roots, square roots, ratios, percents, absolute values, reciprocals, and exponents. In measurement, citizens should know both traditional and metric measures of length, area, volume, weight (or mass), time, and temperature. They should also know how to convert between these measures. In algebra, functional and useful topics can include simple linear equations, plotting graphs of linear equations, slopes, operations in positive and negative integers, and the concept of proportional reasoning. In geometry, citizens should know the various area and circumference formulas for circles, squares, rectangles, and triangles. They should be familiar with the Cartesian coordinate system in two and three dimensions. They should be able to convert size on a scale model or map to actual dimensional size. They should be able to do basic constructions using a compass and straight edge. Citizens should be familiar with three dimensional shapes in terms of finding the volumes and surface areas of shapes like the cone, pyramid, prism, cylinder, and sphere. In statistics, they should be able to find the measure of central tendencies

[9] http://www.stolaf.edu/people/steen/Papers/01case-for-ql.pdf
[10] http://www.beteronderwijsnederland.nl/files/active/0/De%20Lange%20ML%202006.pdf

> when given a set of values. They should be able to graph data as a histogram, pie chart, bar graph, and line graph. In probability, they should know probabilities based on theory and probability based on experiment, compare risk factors of a given situation; calculate the basic probability of outcomes using the multiplication principle, permutations, or combinations.
>
> (Ojose, 2011, p. 97)

Concluding Comment

A study of mathematics at school aims to develop mathematically-literate citizens who can use the knowledge and competencies of mathematics in their daily lives at home and at work. Tracing the understanding of quantitative literacy / numeracy / mathematical literacy is a complex and evolving field. Though considered important, understandings of quantitative literacy or mathematical literacy are somewhat contested. There is also considerable debate on the level of skills and competences needed. Is it sufficient to have a basic knowledge of number sense and algebra to be mathematically literate? Perhaps it is necessary to have minimum skills in arithmetic, measurement, algebra, geometry, probability, statistics and logic to qualify as a mathematically literate person. What, in essence, is required to understand and apply basic mathematics in our everyday lives? The next chapter considers the primary school mathematics curriculum which outlines the pivotal role that mathematical literacy has in equipping a learner for life.

2

—

Mathematics in the Primary School Curriculum

Introduction

This chapter provides an overview of the Primary School Curriculum (1999) for mathematics. Mathematics in the National Strategy for Literacy and Numeracy *Learning for Life* is then considered. The transition from primary to post-primary is briefly mentioned. The chapter also provides an overview of the mathematics curriculum for primary schools in Northern Ireland.

The Primary School Curriculum (Mathematics)

The Primary School Curriculum (1999, p.2) sees mathematics as 'the science of magnitude, number, shape, space, and their relationships and also as a universal language based on symbols and diagrams. It involves the handling (arrangement, analysis, manipulation and communication) of information, the making of predictions and the solving of problems through the use of a language that is both concise and accurate.'[11]

The curriculum outlines the pivotal role mathematical literacy has in equipping a learner for life:

> Mathematics education is concerned with the acquisition, understanding and application of skills. Mathematical literacy is of central importance in providing the child with the necessary skills to live a full life as a child and later as an adult.
>
> (Primary School Curriculum, Teacher Guidelines: Mathematics, 1999, p.2)

The Primary School Curriculum plays a key role in preparing children to meet the demands of the 21st Century. The mathematics curriculum prepares children to think and communicate quantitatively and to use maths to solve problems. Children need to be able to make sense of data encountered in the media, and to be competent in terms of vocational mathematical literacy. As outlined in the curriculum, mathematics education provides the child 'with a wide range of knowledge, skills and related activities that help him/her to develop an understanding of the physical world and social interactions' (p.2). In addition, children learn the language of mathematics which enables them to 'analyse, describe and explain a wide range of experiences, make predictions, and solve problems' (p.2). The mathematics curriculum aims to foster children's creative and aesthetic

[11] http://www.curriculumonline.ie/en/Primary_School_Curriculum/Mathematics/Mathematics_Curriculum/

development. It also seeks to enhance the 'growth of reasoning through the use of investigative techniques in a mathematical context' (p.2). The mathematics curriculum encourages the child 'to be confident and to communicate effectively through the medium of mathematics' (p.2).

The curriculum describes the body of knowledge of mathematics, and the skills and procedures that can be used 'to describe, illustrate and interpret; to predict, and to explain patterns and relationships in *Number, Algebra, Shape and space, Measures and Data'* (p.2). Mathematics has its own language to assist in organising, manipulating and communicating information, both conveying and clarifying meaning. In mathematics children learn to 'explain, predict and record aspects of their physical environments and social interactions' (p.2), thereby enriching their understanding of their world. Contemporary society has been profoundly influenced by the application of increasingly sophisticated mathematics in the fields of economics, technology, science and society. Mathematics is, therefore, an essential tool for both children and adults.

It is important for children to be able to deal effectively with mathematical dimensions of their lives, to think and communicate quantitatively and spatially, to solve problems, to use technology, and to make sense of the 'mass of information and data available through the media' (p.2). The curriculum acknowledges that 'mathematics is an intellectual pursuit in its own right' and that it is 'a source of fascination, challenge, and enjoyment' (p.3). Children are more likely to develop an enthusiasm for the subject when they can explore patterns and relationships, solve problems, develop an appreciation of designs and shapes and when they have an awareness of the historical and cultural influences that have shaped modern mathematics.

Therefore, the primary mathematics curriculum 'seeks to provide the child with a mathematical education that is developmentally appropriate as well as socially relevant' (p.3). It is important that the mathematics programme in each school reflects children's needs and is 'sufficiently flexible to accommodate children of differing levels of ability' (p.3). Through mathematics education children should have 'interesting and meaningful mathematical experiences' and opportunities to 'apply mathematics in other areas of learning', if they are to become 'mathematically literate members of society' (p.3). Integration with all the other subjects adds a valuable perspective to the mathematics curriculum.

The structure of the curriculum

The curriculum comprises five strands:

- Number
- Algebra
- Shape and space
- Measure
- Data.

These strands 'should be seen and taught as interrelated units in which understanding in one area is dependent on, and supportive of, ideas and concepts in other strands' (p.3). Successful teaching of the curriculum requires linkage within the subject. The strands are divided into strand units, which give additional structure to the curriculum. The strand units are described as follows in the curriculum:

> Number starts with a section called early mathematical activities, in which there are four strand units: Classifying, Matching, Comparing and Ordering. These units develop at

infant level to include counting and analysis of number. In first and second classes the development includes place value, operations and fractions. Decimals are introduced in third class and percentages in fifth class.

Algebra is formally recognised at all levels and covers patterns, sequences, number sentences, directed numbers, rules and properties, variables and equations.

Shape and space as a strand explores spatial awareness and its application in real-life situations. It includes units dealing with two-dimensional and three dimensional shapes, symmetry, lines and angles.

Measures consist of six strand units: Length, Area, Weight, Capacity, Time and Money.

Data includes interpreting and understanding visual representation. Chance promotes thinking, discussion and decision-making and is familiar to children in the form of games and sporting activities. (Primary School Mathematics, Mathematics, 1999, pp. 3-4)

All children have some mathematical knowledge and language when they start school, based on their experiences at home and/or in pre-school. Through play, young children experience mathematics when sharing and comparing toys, expressing ideas and learning to co-operate with each other. Children continue to develop their skills through structured play and early mathematical activities in school. In infant classes, children develop their confidence in handling materials and in using mathematical language.

Across all content of the curriculum children should develop the following: applying and problem-solving, communicating and expressing, integrating and connecting, reasoning and implementing, understanding and recalling. Children also have opportunities 'to explore the nature of mathematics and to acquire the knowledge, concepts and skills required for everyday living and for use in other subject areas' (p. 4).

Table 1: ***The Mathematics Curriculum in Junior and Senior Infants***

Strands	**Strand Units**
Early Mathematical Activities	• Classifying • Matching • Comparing • Ordering
Number	• Counting • Comparing and Ordering • Analysis of number combining, partitioning, numeration
Algebra	• Extending patterns
Shape and Space	• Spatial awareness • 3D Shapes • 2D Shapes
Measures	• Length • Weight • Capacity • Time • Money
Data	• Recognising and interpreting data

(Primary School Curriculum, Mathematics, p. 17).

Number is always presented first in the curriculum documents. However, strands can be taught in parallel rather than one after the other and aspects of number are taught throughout the mathematical curriculum. The curriculum is presented in the same way for the rest of the primary curriculum i.e. first/second class, third/fourth class and fifth/sixth class.

In implementing the curriculum teachers take children's individual differences into account. Continuous assessment is encouraged. Teachers identify children's knowledge and where they might be having difficulties, and differentiate their planning to reflect the needs of their pupils. A constructivist approach to mathematics is used, involving pupils as active participants in their learning. Experimentation and discussion play a key role in mathematics teaching, allowing pupils to construct meaning and to solve problems under the guidance of the teacher. The curriculum emphasises the importance of mathematical language. Words and phrases can have different meanings in mathematics. Children need to develop their understanding of mathematical concepts and be able to express mathematical ideas. Discussion is central to this process. Mathematical symbols are also taught, but it is important that they are not taught prematurely.

Practical experience in using and manipulating mathematical equipment is emphasised in the curriculum, enabling children to develop their understanding of mathematics based on experiences and interactions with the environment. There is a greater emphasis on mental calculations, estimation, and problem-solving skills and less emphasis on long, complex, written calculations. Calculators are used from 4th to 6th class to support the development of problem-solving skills, to assist in checking estimates, to perform long and complex calculations and to check solutions. Calculators support the development and mastery of number. Computers also have a place in teaching mathematics, and are a valuable tool for both teachers and pupils. Computers can be stimulating for less able children and can provide extension work for different levels of ability. However, the emphasis must always be on the process, for example, collecting information, deciding on the relevance of questions, and interpreting results.

Problem-solving is a core element of mathematics which fosters higher-order thinking skills. Problem-solving facilitates discussion and cooperative working, enabling children to 'to analyse mathematical situations; to plan, monitor and evaluate solutions; to apply strategies; and to demonstrate creativity and self-reliance in using mathematics' (p. 8). Children develop confidence through solving problems successfully and can see how mathematics is used in their day to day lives. Children meet mathematics in most areas of their lives. Therefore, mathematics is integrated with other aspects of the curriculum. For example:

> SESE provides ample opportunities for using mathematics, for example recording results of experiments in science or creating maps in geography, while a sense of time and chronology is essential in history. Collecting data for analysis is also an important feature of SESE and provides the child with real-life examples of data with which to work. Physical education offers myriad opportunities for measurement as a natural part of the activities, for example timing races or measuring the length of jumps. Creating symmetrical and asymmetrical shapes in a gymnastics lesson can also offer real use of mathematical concepts. Mathematical language occurs in all areas of the curriculum, for example in long and short notes in music or using the correct words to describe shapes in visual art activities. Many teachers make use of rhymes, songs and games to reinforce concepts of number and shape, and this can be achieved in English, Irish or using a modern European language where appropriate.
>
> (Primary School Curriculum: Mathematics, 1999, pp. 8-9)

Mathematical literacy is of central importance in providing children with the necessary skills to live a full life as children and later as adults. It is important for society that people can think and communicate quantitatively. It is also important that situations where mathematics can be applied to solve problems are recognised. As stated in the curriculum, 'it is necessary to make sense of data encountered in the media, to be competent in terms of vocational mathematical literacy and to use appropriate technology to support such applications' (Teacher Guidelines, p.2). The mathematics curriculum, which is concerned with the acquisition, understanding and application of skills is a key factor in preparing children to meet the demands of the 21st century.

Assessment

Assessment is an integral part of teaching. According to the Primary School Curriculum, assessment is a "continuous, dynamic and often informal process" (p. 114). Assessment includes classroom observation, questioning and dialogue, structured tests by teachers and standardised tests. Assessment informs teachers' teaching by providing information which teachers can use to plan to meet their pupils' needs. However, assessment is not an end in itself, therefore it is important that a balance be struck between the time spent on assessment and the time spent on teaching. The constructivist approach to mathematics facilitates the gathering of assessment information. Assessment should be a positive experience for the child, with teachers helping children to develop their own learning.

Assessment has a formative role, providing feedback on children's learning, identifying their strengths and weaknesses, and assisting in identifying where pupils may be experiencing difficulties in developing mathematical concepts and skills. Diagnostic assessment helps to identify children with difficulties in specific areas of mathematics and children with special educational needs. Assessment assists teachers with planning and in examining the suitability of the curriculum content and methodologies and approaches in their own class. Assessment is also summative, where children's progress is recorded at particular times, such as the end of the year, or at the end of teaching a particular topic. Summative assessment is useful when reporting to parents and in providing information for other teachers.

The main emphasis of assessment is to focus on what children know, what they can do and how they do it. Both the process and product of children's learning are important. Cognitive and affective areas of children's learning should be assessed.
The areas that should be assessed in mathematics are outlined in the Primary School Curriculum, as follows:

- conceptual knowledge and understanding is assessed in the application of mathematical concepts, for example the conservation of length. This must be done in a variety of contexts, including observation of the child performing a task or noting the child's application of a concept in a real-life situation
- problem-solving ability is assessed by evaluating the approaches, strategies and processes the child uses in dealing with mathematical tasks and the connections he/she makes within mathematics itself and within other subjects
- computational proficiency includes assessing the use of number, the appropriate application of the four number operations, and the ability to compute numbers efficiently, both mentally and in written situations
- recall skills are assessed in the recall of number facts, terminology, definitions and formulae and in their efficient use within a given situation. This is a particularly important skill in the area of estimation

- mastery of specific content areas (for example number, algebra, measures, shape and space, data) is assessed through the application of these areas in practical, everyday contexts
- the ability to communicate and express mathematical ideas and processes and the correct use of mathematical language in oral and written form can be assessed by observation while the children are engaged in a mathematical task. Discussion of their own work can reveal gaps in their knowledge and skills. Incomplete understanding of mathematical terminology or processes can also be identified. When recording, children can communicate pictorially, orally or in written form using words and/or symbols
- attitudes towards mathematics, including confidence, interest, willingness to take risks, and perception of the usefulness of mathematics, are assessed by observing the enthusiasm with which the child approaches a task. Attitudes also encompass the interest the child shows in completing tasks and in using mathematics confidently in other curricular areas and in real-life situations. Teachers' observations of such attitudes contribute to an overall picture of the child's mathematical development and are continuing and informal.

(Primary School Curriculum: Mathematics, 1999, pp. 115-116)

A broad range of assessment tools is available in mathematics, including portfolios of children’s work, observation records, mastery checklist results, teacher-designed tests and standardised tests.

Teachers observe their pupils every day as they engage in mathematical activities, noting where particular children may be having difficulties, and using their observations to plan their next approach or lesson. Discussing their work with children or analysing their written work often indicates how a child might be thinking, what processes they are using and what gaps in knowledge or skill might exist. Teachers use this information to plan. Teachers design their own assessment tasks and tests to help understand children’s progress and which also provide useful information which they can use in their planning. Teacher-designed tasks and tests are usually closely linked to the instructional objectives of particular lessons and provide formative, diagnostic and summative data on children's progress.

Samples of children’s work may be kept in a portfolio, a folder or a file, providing a record of development over a year. The samples of children’s work contained in the portfolio can provide the basis for discussions of pupils’ work with both children and their parents and can be passed on to other teachers. Curriculum profiles are useful in enabling teachers to make an overall judgement of pupils’ achievements, as they allow for an interpretation of a wide span of outcomes. Early screening tests and diagnostic tests assist in identifying children experiencing difficulties with aspects of mathematics, enabling teachers to develop intervention strategies and learning support to particular pupils. Tests can also identify where children need extension work.

Standardised tests have become a stronger feature of primary education since the introduction of the Primary School Curriculum in 1999. It is now obligatory for schools to carry out standardised tests at the end of 2nd, 4th and 6th class and to forward their results to the DES. The Primary School Curriculum describes standardised testing as follows:

> Standardised tests comprise norm-referenced tests and criterion-referenced tests. Norm-referenced tests compare pupils with other pupils or with national standards. They consist of highly structured tasks that have associated with them a set of scoring rules.

Standardisation refers to the uniformity of procedures in administering a test. All children take the same test under the same time limits and instructions. These rules must be adhered to rigidly in order to produce a standard score and maintain the validity of the test. Administering the same test to all children under the same conditions means that achievement can be judged independently of external factors.

(Primary School Curriculum: Mathematics, 1999, p. 119)

Maths in the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2012

The Department of Education and Skills published a national strategy to improve literacy and numeracy among young people in July 2011, called *Literacy and Numeracy for Learning and Life*. In the press release that accompanied the launch, Minister Quinn stated:

> It is the government's belief that no child should leave school unable to read and write and use mathematics to solve problems. We know that there is currently much room for improvement and this strategy sets out the road map with concrete targets and reforms that will ensure our children, from early childhood to the end of second level, master these key skills.
>
> (Minister for Education and Skills, 8 July, 2011)

According to the Press release "the Strategy aims to ensure that teachers and schools maintain a strong focus on literacy and numeracy skills, within a broad and balanced curriculum". The Government set the following targets in the strategy to be achieved by 2020:

- increasing the number of children performing at Level 3 or above (the highest levels) in the national assessments of reading and mathematics by 5 percentage points, at at primary level
- reducing the percentage performing at or below the lowest level (Level 1) by 5 percentage points
- increasing, at post-primary level, the number of 15-year old students performing at Level 4 or above (the highest levels) in the OECD's PISA test of literacy and mathematics by at least 5 percentage points
- halving the numbers performing at Level 1 (the lowest level) in PISA test of literacy and mathematics
- improving early childhood education and public attitudes to reading and mathematics.

(DES, 2011)

The publication of the Strategy was followed by the issuing of Circulars to schools requiring them to increase the time available for literacy to 90 minutes per day and for mathematics to 50 minutes per day (up from 35 minutes currently). The aim is to ensure that every child leaves school having mastered literacy and numeracy.

Background

It is useful to consider the background to this Strategy. In November 2010, the Department of Education and Skills published a draft national plan for improving literacy and numeracy and asked for comments from the education community and the public. The publication prompted a wide debate about literacy and numeracy. A large body of individuals and groups – some of whom were directly involved in or with the education system and some who were not - submitted detailed comments and suggestions to the DES on how to best develop and fine-tune the strategy. This included a detailed and considered submission by the INTO on behalf of its members – this submission was informed by consultative meetings, on-line fora, general feedback from members and the work of a specially-convened task group. The INTO submission is available on www.into.ie. Together with other submissions from a variety of groups and individuals, it is also available on www.education.ie. Additionally, officials from the Department met with over sixty organisations to hear more about their suggestions and to examine the key proposals that they had made – this included a meeting with INTO representatives.

Numeracy – a definition

In his foreword to the Literacy and Numeracy Strategy, Minister Quinn emphasised the nature of the Mathematics that is to receive focus. He stated that it is important that children and young people in the education system will be "able to understand and use mathematics in their everyday lives and in further learning" (DES, 2011, p.5). The concept of numeracy, which underpins the Strategy, is defined as follows:

> **Numeracy** is not limited to the ability to use numbers, to add, subtract, multiply and divide. **Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings.** To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.' (original emphases).
>
> (DES, 2011, p. 8)

The importance of numeracy in terms of the young person's personal benefit and life chances and in terms of the greater good of society and the economy receives repeated mention throughout the Strategy.

The School and Beyond

The Strategy acknowledges that students can fail to develop adequate literacy and numeracy skills for various reasons, some of which "lie beyond the school and education system" (DES, 2011, p.9) – reasons such as those rooted in socioeconomic factors. Nevertheless, it also asserts that "effective schools and educational interventions can improve learning outcomes substantially for all students, including those from disadvantaged backgrounds" (p.10). While the Strategy also makes reference to how groups and individuals beyond the school can have a positive impact on the numeracy of children and young people, its predominant focus is on the role of the education system - from early childhood care and education (ECCE) through to primary and second-level schools - in providing the best possible opportunities for young people to acquire good numeracy skills. Continuity across the various stages of a child's education is stressed and the strategy emphasises the importance of a "seamless transition for [the child] at each point of change in the education system" (p.11).

While the strategy acknowledges that "many students in our education system achieve very good standards of ...numeracy", it contends that "a significant minority do not" and that "many students acquire adequate skills but could do even better" (p.12). It refers to matters such as the proportion of students taking Higher Level Mathematics in the Leaving Certificate examination and assessments such as PISA 2009 to back up this assertion.

The document proceeds to state that improving numeracy standards is an urgent national priority, and outlines key areas to which it intends to direct significant attention. These include: setting challenging and clear goals around numeracy learning in all stages of education from ECCE to post-primary, heightened emphasis on continuous improvement, assessment and reporting in all of those settings, enhancing the professional skills related to the teaching of numeracy for teachers of pupils/students in those settings, building the capacity of school leadership to enhance the quality of teaching and learning of numeracy, ongoing focus on boosting opportunities for numeracy learning for those with additional learning needs and those from the most disadvantaged communities, and helping parents and communities and agencies beyond schools to promote and support children's numeracy development (p. 14). The strategy document acknowledges that such goals are ambitious, particularly in time of economic difficulties. It nevertheless prioritizes such goals above others and indicates intentions to put in place detailed plans to realise those targets.

While the focus here is on the Literacy and Numeracy strategy as it relates to the primary school, it is nevertheless appropriate to make some reference to elements of the strategy that are specifically focused on other agencies/bodies/groups when the strategy targets and approaches for those groups contain direct reference or links to the primary school.

For example, significant emphasis is placed on heightening the role of parents in developing their children's numeracy skills. This includes generating higher levels of engagement by parents with their child's primary school. There is an expectation that provision for such engagement would be a feature of the School Plan. The nature of this 'engagement' is quite broad, ranging from, for example, broad-based engagement such as encouraging parents to ensure that their child attends school each day to more specific curricular-related issues such as enabling parents to understand school reports that relate to their child. The strategy also plans to involve schools in "parental and wider community initiatives that promote and support the acquisition of … numeracy skills (e.g., Shared reading, Maths for Fun, library services, etc.)" (p.24). Throughout the strategy, there is much emphasis on linkage and continuity, not only at transition points from ECCE settings to primary school, and from primary school to second-level school but also between and across school settings, home, and other agencies, settings and groups that have or potentially have an impact on the child's numeracy learning – there is also an aspiration that these various agencies and settings would share and mutually enhance their professional expertise in this area.

The strategy acknowledges the key role that teachers play in developing numeracy skills in pupils/students. It stresses the necessity of recruiting students of high calibre into initial teacher education (ITE) courses, and emphasises the need for commitment to quality professional education and development right throughout the teacher education continuum. Specific reference is made to 'equipping' teachers to develop the numeracy skills of the pupils/students with whom they work – to help them to develop a deeper conceptual understanding in mathematics and the ability to apply mathematical concepts in real-life scenarios.

Primary and Transition to Post-Primary

The NCCA has prepared a 'Bridging Framework' to support the transition from primary school to post-primary school in relation to teaching and learning mathematics. The post-primary mathematics curriculum has undergone revision in recent years through Project Maths. Building on mathematics at primary level, a more investigative approach will be used in the Junior Cycle. The Bridging Framework links the various strands of mathematics in the primary school to topics in the Junior Certificate mathematics syllabuses in order to ensure better continuity, and highlights for parents and pupils how the objectives of the Mathematics Curriculum at primary level are continued and progressed at post-primary level.

Project Maths

Project Maths[12] is a programme of reform of mathematics in second level schools. The programme promotes an understanding of mathematics so that pupils can appreciate the relevance of what they are learning, how mathematics can be used to solve problems and how mathematics is applied to everyday life. Project Maths involved a change in the mathematics syllabus at both Junior and Senior level both in terms of how mathematics is assessed and how mathematics is taught and learnt. In Project Maths there should be a better balance between understanding mathematical theory and concepts and developing practical applications skills. Project Maths involves a cultural change in how mathematics is taught at post-primary level. The initiative aims to improve mathematical standards and broaden the understanding of what mathematics learning encompasses.

Project Maths involves changes to what students learn in mathematics, how they learn it and how they will be assessed. The initiative aims to provide for an enhanced student learning experience and greater levels of achievement for all. Greater emphasis is placed on the understanding of mathematical concepts, using contexts and applications that enable pupils to relate mathematics to everyday experience. Project Maths also has a focus on developing pupils' problem-solving skills. Assessment reflects the different emphasis on understanding and skills in the teaching and learning of mathematics.

Project Maths was initially piloted in 24 schools, which were centrally involved in curriculum development for the Project. The Mathematics syllabuses include the following strands:

- Statistics and Probability
- Geometry and Trigonometry
- Number
- Algebra
- Functions

According to the NCCA, Project Maths builds on existing knowledge and skills, promotes progression and continuity, moves from the concrete to the abstract and from the informal to the formal. Project Maths seeks to develop learner confidence and competence, it focuses on problem-solving skills and strategies and students learn to apply knowledge and skills to familiar and unfamiliar contexts and problems (Project Maths – improving understanding - Bill Lynch, NCCA, Mathematics Symposium – 2 Feb 2010). Project Maths aims to address the perceived negative attitudes to mathematics among students as identified in *Inside Classrooms – The teaching and learning of mathematics in social context* (2003).

Mathematics Curriculum in Northern Ireland

In light of proposed changes to the curriculum in the 26 counties, it might be useful to consider numeracy within the context of the new statutory framework for the curriculum in Northern Ireland.

The Education Reform Order of 1989, which was the forerunner to the new Statutory Assessment Arrangements 2012/13, formally put arrangements in place that enabled teachers across Great Britain and Northern Ireland to draw up standardised levels of

[12] http://www.projectmaths.ie/overview/

numeracy performance at three different stages of a child's life at school. This was to be divided into four key stages (KS) and the statutory assessments were to be carried out at the end of the first key stage (Year 4) when the child had reached seven, at the end of the second key stage (Year 7) when the child had reached eleven and at the end of the third key stage (Year 10) when the child had reached fourteen. Year 12 pupils [aged 16] would continue with the GCSE examinations.

Numeracy or mathematics, being one of the 'compulsory' subjects (Department of Education Statutory Order), was a prescribed programme of study with attainment targets. The programme of study detailed the skills and processes which needed to be covered and the parallel attainment targets described the range of knowledge, understanding and skills needed or attained by the pupil for successful progression to the next attainment target. Teachers were initially provided with INSET training (inservice) to familiarise themselves with the programme of study, in order to judge and standardise pupils' work and then measure it against the level descriptors.

Pupils at the end of KS1 and KS2 [Primary School] were formally assessed by teachers in English and mathematics. Science was added as the third formal assessment at post-primary level at KS3. Originally assessment in school years four and seven took the form of using both moderated teacher components and Mandatory Assessment Units. Assessment Units (AU) were external short, informed tasks supplied by Council for the Curriculum Examinations and Assessment (CCEA) and were used as part of normal class work. A measure of subjectivity could still be present as the AUs, at Key Stages 1 and 2, were marked internally by classroom teachers. Importantly the results were reported to parents. The Assessment Units were meant to confirm the teachers' level placement of pupils.

The overall numeracy level was arrived at by combining the teacher's assessment of a pupil's work in the following subject-related attainment targets:

- Processes in Mathematics
- Number
- Measures
- Shape and Space
- Handling data

The teacher examined the child's work over the length of the Key Stage and made judgements, using the level descriptors from each attainment target. The Assessment Units were then used to confirm their judgements.

Schools were regularly requested to submit to CCEA[13] varied samples of work, from different levels, to enable their school to be compared with other schools. At the time it was felt that the majority of pupils were, at the end of KS1, working at level 2 and, at the end of KS2, working at level 3-4. It was anticipated that the new Levels of Progression would be more demanding than the previous Levels of Attainment. However, in some circumstances, some children could achieve higher than the expected level; a level 3 could possibly be achieved at KS1 and a level 5 could be awarded at KS2.

The reform order of 1989 was seen by teachers in Northern Ireland as one of the greatest changes to their methodology of teaching and learning. Over the subsequent years, major research carried out on behalf of the Department of Education led to changes and amendments, culminating in the most recent major review, the new Statutory

[13] Council for Curriculum, Examinations and Assessment

Assessment Arrangements. From September 2012, new assessment arrangements were put into place to support the Northern Ireland Curriculum. This review means that from 2012/13 mathematics will be assessed through the Key Stages with reference to the new Levels of Progression, which have replaced the earlier Levels of Attainment. While teachers will no longer use AUs and tests at Key Stages 1, 2 and 3, they will still produce summative judgements to report to parents at the end of each Key Stage. The rationale is that there will be a greater emphasis on mapping numeracy to Cross Curricular Skills:

> At the heart of the curriculum lies an explicit emphasis on the development of success and personal capabilities for lifelong learning and for operating effectively in society.
>
> (The Northern Ireland Curriculum: Primary, KS1&2, 2007, p. 1)

The new Statutory Assessment Arrangements rely on external moderation. Similar to earlier assessment projects, teachers will still need to create conditions where standards can be monitored for consistency. One new aspect of this 'quality control' is the need to have appropriate procedures in place for schools that do not apply standards consistently. The moderation process means that schools will submit the awarded levels for all pupils at the end of each Key Stage and CCEA will scrutinise the data sent by the school, before requesting specific pupil portfolios. The sample requested will depend on the school size. CCEA will then examine the requested portfolios to confirm appropriate and accurate administration of awards and then feedback from CCEA will indicate if schools have properly satisfied the award criteria and if adjustment is needed or not.

The Northern Ireland Curriculum acknowledges that numeracy is the development of mathematics across the curriculum and skills acquired by pupils from the use of numeracy should help them make informed choices throughout their lives. Pupils will, throughout primary school, engage in activities that include:

- Playing
- Exploring and investigating
- Doing and observing
- Talking and listening
- Questioning
- Reflecting
- Drafting
- Reading and recording

(Northern Ireland Curriculum: Primary, 2007, page 57)

The emphasis on the use of Cross Curricular Skills to measure pupil competence is encouraged further so that teachers will guide pupils to develop and then assess the core skill of using mathematics across the entire curriculum. This will be achieved by exposing pupils to a wide range of opportunities for the acquisition and development of Cross Curricular Skills.

There are two aspects to numeracy and mathematics in the new Levels of Progression. These are **Requirements** and the **Knowledge & Understanding**. The Levels of Progression lays out a progression of acquired skills in the form of 'can do' statements. Elements of the **Requirements** across the curriculum include the following:

- Using Mathematics Knowledge and concepts accurately
- Using Mathematics to solve problems and make decisions

- Identifying and collecting information
- Using ICT to solve problems and/or present work

To demonstrate skills in using mathematics pupils should use **knowledge & understanding** of the following:

- Number
- Measures
- Shape & Space
- Handling data

Thereby the **Requirements** identify how pupils are able to use their **knowledge & understanding** across the curriculum to

- Communicate
- Manage information
- Think critically
- Problem solve
- Make decisions

The **Knowledge & Understanding** component focuses on mathematics that pupils can use across the curriculum, and the **Requirements** use skills that are generally generic across the curriculum whereby they cannot be viewed in isolation. However, teachers must address both the **Requirements** and the **Knowledge & Understanding** components of *Using Mathematics,* when determining the level of progression a pupil is working at.

The use of Assessment for Learning Strategies and Personal Capabilities in classroom practice is demonstrated by the following Traffic Survey exemplar. This activity provides an opportunity to connect pupils' learning by integrating the **Requirements** with **Knowledge & Understanding** for using mathematics.

Group Traffic Survey Activity
Pupils work in pairs or small groups to complete a traffic survey based on vehicles that pass a particular point at or near the school. In their groups, pupils will gather the information of the passing traffic and then present it by drawing and labelling the axes and completing a bar chart of their findings. This activity connects pupil learning by integrating the following **Requirements** of using mathematics:

- Using mathematical understanding and language to ask and answer questions, talk about and discuss ideas and explain ways of working
- Solving problems and making decisions
- Reading, interpreting, organising and presenting information in mathematical formats

Pupils also used **Knowledge & Understanding** to

- Collect, group, record and present data
- Interpret and present data using a bar chart
- Add, subtract, divide and multiply whole numbers

A real life context is also experienced by the pupils taking part in this exercise and they are able to use the Thinking Skills and Personal Capabilities of

- Managing information
- Thinking, Problem-Solving and decision making
- Working with others

The task could be expanded and adapted to include the Thinking Skills and Personal Capability of Self-Management.

However, teachers have raised concerns regarding the new Statutory Assessment Arrangements. While it is generally accepted by all of the teaching unions that the new assessment arrangements have led to increased bureaucracy and workload, teachers complained that there was confusion over the implementation of the arrangements, especially assessment tasks and deadlines for submissions. Teachers in schools not participating in voluntary moderation were unsure of their requirements. There were problems associated with the use of pupil portfolios as evidence submitted for moderation. Many INTO teachers felt disadvantaged by the deadlines of the moderation process because many of them had participated in the industrial action directive from December 2011 which stated that they were not to participate in the new assessment arrangements, and therefore had not attended important aspects of training.

Furthermore, the greatest area of concern was the ICT aspect of assessment. There were problems of reliability and validity with ICT testing; levels obtained through ICT did not correspond with the Levels pupils obtained through National Federation for Educational Research (NFER) Standardised tests. This became so fraught with difficulty that the INTO described it as a 'shambles' and in October 2012 INTO members were advised that if they had any technical or operational concerns, they should immediately stop using ICT tests.

Regarding numeracy and the new assessment arrangements, it was clear that all stakeholders were not completely in agreement. The debate regarding the future provision of education in Northern Ireland continues.

Concluding Comment

The importance of mathematical literacy is highlighted in the Primary School Curriculum (DES, 1999) and endorsed in the National Strategy for Literacy and Numeracy (DES, 2011). The use of mathematics in every-day situations was emphasised by the NCCA in its support guide for parents (Helping your young child with Maths[14]), aimed at encouraging parents to support the mathematical development of their children. The National Strategy to Improve Literacy and Numeracy among Children and Young People has as its aim that every child should leave school having mastered literacy and numeracy and sets out measures to improve numeracy, including increasing the amount of time spent teaching mathematics and setting targets for national achievement in international assessments like OECD's PISA. Mathematics education is about understanding concepts and theories and their applications to real life. A revised primary curriculum will build on the Primary School Curriculum of 1999, taking on board feedback on the curriculum to date, and progress in national and international assessments and the Literacy and Numeracy Strategy. Lessons can also be learned from developments in Northern Ireland.

[14] http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/Primary/Infant_classes_Maths_resources/

3

—

Numeracy: A Glance at Some Reports / National and International Assessments

Introduction

Nowadays numeracy is not viewed as just the ability to use numbers. Rather it is considered as being the wider ability to use mathematics to solve problems and meet the demands of day-to-day living in complex social settings. Having young people who can apply mathematical understanding in a growing range of economic, technical, scientific, social and other contexts is essential if we are to provide employment and economic prosperity in the future (DES, 2011). Therefore, numeracy skills could be considered as essential for every young person as literacy skills are. The basic skills of literacy and numeracy are used in practically all aspects of daily living, for example, when we communicate with each other in traditional forms or through the web, when we follow signs and instructions, at work or in leisure, when we try to understand information and data available through the media and when we are managing our lives (DES, 2011).

For these reasons the DES considers it essential that every child leaving the Irish school system will be literate and numerate - able to speak, read, write and spell at a level that enables them to communicate and participate fully in education and in Irish life and society. Much of this learning takes place in the primary school but according to the DES (2010) critical aspects of the child's development on literacy and numeracy skills continue during the compulsory period of second level education.

In general, the centrality of mathematics education is not underestimated. Indeed, it is perceived as a core area of learning in probably most education systems throughout the world. Educational research has sought to compare achievement in mathematics between various countries and Ireland has participated in a number of national and international assessment studies. According to Greaney and Kellaghan (1996) the analysis of such assessments is used to inform policy, monitor standards, introduce realistic standards, promote accountability, increase public awareness, direct teachers' efforts towards raising pupil achievement and inform public debate. Some of these assessments, studies and surveys carried out in recent years will be presented, examined, and reviewed in this chapter. There have been evaluations carried out also which were specific to DEIS[15] schools in recent years and these will also be considered.

[15] Delivering Equality of Opportunity In Schools

Influences on mathematical achievement

Growing up in Ireland (GUI) (ESRI, 2012) is a national longitudinal study of children that examined the home, school and community influences on nine-year olds' learning. As part of that research the relationship between participation in out-of-school activities and children's academic achievement were examined based on Drumcondra reading and maths assessments. One significant finding - that school and classroom factors made more of a difference to mathematics scores than any other factor, is noteworthy; this is in keeping with international research which shows that mathematical learning is strongly associated with the school while literacy skills are often developed and reinforced in the home as well as in school. Mathematics scores did not vary by language medium of the school, gender mix or location controlling for pupil characteristics. As with reading there was a significant achievement gap between those attending urban DEIS schools and other students. Children achieved higher scores in mathematics when classes were taught by more experienced teachers and where there was access to the internet. Interestingly though, the GUI 2012 study found that the presence of a computer in the classroom was not associated with higher mathematical performance. This concurs with US research on outcomes from the National Assessments of Educational Progress which suggest that teacher qualification and experience are the strongest correlates of reading and mathematics achievement, both before and after controlling for student poverty and language status (Darling-Hammond, 2000).

As with reading, significant differences in Mathematics performance were evident by children's out-of-school activities, even when other factors were taken into account. Overall, the findings confirm that involvement in cultural activities is associated with higher educational performance. Insights into the nature and impact of children's out-of-school experiences and informal learning can be gleaned from Bourdieu's socio-cultural reproductive theory ((Bourdieu, 1984). This perspective focuses on the unequal distribution of economic, social and cultural resources (or capital) across classes and their transmission from parents to children. Being familiar with the dominant culture is a form of cultural capital and influences the individual's predispositions to learning, among other factors (Reay, 2004). Middle class parents thus possess the kinds of cultural and social capitals that enable them to foster their children's educational success. The social network group in the GUI 2012 study had an advantage relative to the TV/Sports group, reinforcing that the kinds of unstructured activities engaged in also play a part in shaping educational outcomes. This too concurs with Bourdieu's theory and findings: for example, the sports/computer games group achieved higher mathematics scores than the TV /sports group: this could indicate the potential importance of ICT usage in academic achievement. In contrast to the finding in reading, children leading 'busy lives' had a slight performance advantage over the TV/sports group; it may be that literacy skills are squeezed out by over involvement in recreational activities because they rely on more positive reinforcement outside school than mathematics. It is clear, therefore, that school plays a greater role in shaping mathematics performance than in reading. The DES states that numeracy is developed mainly but by no means exclusively through the teaching of mathematics in primary and post-primary schools (DES, 2012). However, the achievement gains associated with participation levels in out-of-school activities in the GUI 2012 study were of similar magnitude for reading and mathematics: reflecting, perhaps, the enhancement of a wide range of cognitive skills in addition to literacy.

International Reports PISA and TIMMS.

The Programme for International Student Assessment (PISA), a project sponsored by the Organisation for Economic Co-operation and Development (OECD), assesses the knowledge and skills of 15-year-olds in reading, mathematics and science. It takes place in three-year cycles, the first of which was 2000. Subsequent cycles took place in 2003, 2006, 2009 and 2012. In each cycle, one of the subject areas is designated the main focus of the assessment. In the first cycle reading was the main focus, in 2003 it was mathematics and in 2006 science. Reading became the main focus again in 2009 while mathematics was a major domain in 2012. Students in 65 countries (including all 33 OECD member states) participated in PISA 2009, which was implemented in March/April in Ireland. A consequence of mathematics being a minor assessment domain in 2009 is that mathematics items appeared in only 9 of the 13 test booklets, and mathematics scores were inputted for students not attempting mathematics items, so that each student in the PISA assessment was assigned a mathematics score whether they were asked to complete mathematics questions or not. All 35 mathematics items administered in PISA 2009 had been administered in earlier PISA assessments, though individual students asked to solve mathematic problems encountered a subset of the 35.

Historically, students in Ireland have not performed well in international assessments of mathematics achievement, though primary level pupils have done somewhat better than their post-primary counterparts (Sheils, Moran, Cosgrave & Perkins, 2010). In an analysis of Trends in International maths and Science Study (TIMMS) 1995 data by the OECD, students in fourth class in Ireland achieved a mean score that was above the OECD average, whereas the mean score of students in Second Year was not significantly different from the OECD mean (OECD, 2000). In the first three cycles of PISA, 15-year-old students in Ireland achieved mean scores in mathematics that were not significantly different from the OECD country average scores. In PISA 2000, students in Ireland did not do particularly well in problem solving, or space and shape, with girls, in particular, performing relatively poorly in that domain (Cosgrove et al., 2005). According to Shiel et al., (2010), there is relatively little available corroborating evidence to indicate that standards in mathematics among students in post-primary schools in Ireland have declined to the extent reported by the OECD in recent years. Between 2003 and 2009, the percentage of Higher-level A-C grades awarded in Leaving Certificate mathematics ranged from 75.6% in 2005 to 79% in 2008, with 77.6% achieving these grades in 2009. However, in PISA 2009 Ireland's average score in mathematics dropped by 16 points since 2003 – it was estimated then that one in five (21%) students in Ireland had performed below the OECDs benchmark Level 2 in mathematics. The Educational Research Centre (ERC) analysed these assessments and the results suggest that declines in both proficiency and engagement may have been at play based on evidence in the increase in the number of questions that students did not attempt (Press Release, ERC, April, 2012).

Interestingly though, much of the media commentary on examination results in mathematics in recent years, has focussed on the proportions of students who do not achieve a grade D or higher in the Leaving Certificate. In Ireland, 20.8% achieved at or below Level 1 on the mathematics proficiency scale, compared with an OECD average of 22%. However, only 6.7% achieved Level 5 or over, compared with an OECD average of 12.7%. There was a greater decline in performance between 2004 and 2009 among higher performers than among lower performers. Even when the 2009 results are set aside, as we seek corroborating evidence of a decline in performance elsewhere, it is clear higher-achieving students in Ireland do not perform particularly well in mathematics. For example, in PISA 2006, 10.2% of students in Ireland achieved Level 5 or higher, compared to an OECD average of 13.3%. Hence, it would seem that there is an urgent need to address the apparent under-achievement of higher performers.

Numeracy in Irish schools: Teaching and learning

Based on survey and assessment results the teaching and learning of mathematics in Ireland requires even greater attention than literacy. As we have seen, surveys of mathematics achievements in the State examinations and in international surveys, indicate that there are systemic issues in mathematics education that require attention. Repeated assessments of primary mathematics revealed weak performance in the areas of problem solving and measures (DES, 2011). At Leaving Certificate level the proportion of students opting for higher level has been in the region of 16% for a number of years, despite an aspiration in the design of the Leaving Certificate mathematics syllabus to have 30% of the cohort taking the higher-level examination. Efforts are now being made to attract students towards opting for higher-level Leaving Cert maths and in 2012 extra points were awarded to those who took the higher paper with a consequential rise in the numbers opting to take the higher-level examination. Another radical change in the curriculum, assessment, teaching and learning of mathematics at post-primary level has taken place with the introduction in post-primary schools of the Project Maths initiative, mentioned in the previous chapter. While it is acknowledged that Project Maths is designed to improve the mathematics performance of students in Ireland, and could, over time, lead to an increase in performance on assessments such as PISA, nevertheless according to Shiel et al., (2010) there are some issues that need to be considered in that regard.

For example, the outcomes of national assessments of mathematics at primary level suggest that some students enter post-primary schooling with relatively low skills in the related areas of problem-solving and measures. Hence, it may be necessary to establish a stronger mathematical basis among primary school pupils so they can fully benefit from Project Maths at post-primary level. Ireland's participation in the mathematics component of the Trends in International Mathematics and Science Study (TIMSS) in 2011 as well as PISA 2012 when mathematics was the main domain for assessment will provide further evidence relating to the performance of primary-level pupils in Ireland. Furthermore, since Project maths is a significant departure from how maths has been taught over decades in post-primary schools, it would seem important to monitor the implementation of the Project carefully, and make adjustments both to content and to teaching approaches as needed. There is also a danger that, in seeking to increase the proportions of students taking higher-level mathematics courses, there will be an insufficient emphasis on ensuring that the performance of the highest-achieving students is enhanced. The question must also be posed: can it be assumed that Project Maths on its own is capable of bringing about the level of change needed to address gaps in the performance of students in Ireland or might there be a need to consider other parallel initiatives to enhance performance? According to the DES (2011) other system-wide measures are needed to improve the way that students engage with mathematics and develop numeracy skills.

School and classroom characteristics

Most, if not all, assessment reports attribute significant importance to the centrality of the role of the teacher in relation to achievement in mathematics. It has been found that the level of qualifications of teachers as well as teacher experience both impact on pupil achievement. Teachers who engage in regular professional development (Eivers et al. 2005), who report high levels of satisfaction with CPD attended (Shiel et al., 2006), who regularly assess pupils with a variety of methods (Eivers et al., 2005), or who use formative assessment (Black & Wiliam, 1998) are those whose pupils tend to display superior achievement.

According to Hiebert and Grouws (2007) teaching mathematics is complex - involving the interaction of student, teacher, curriculum, classroom environment and resources available - suggesting that different teaching methods are effective for different goals, particularly in multi-lesson sequences: all hinging on the effectiveness of the teacher. For example, expository teaching methods may be adequate for achieving skill efficiency but discovery teaching methods are more effective for developing conceptual understanding and making connections.

In Ireland, Delaney (2010) assessed the mathematical knowledge for teaching of 500 primary teachers and found that Irish teachers demonstrate good knowledge of algebra, fractions and identification of pupils' errors. However, he indicated that teachers found applying properties of shapes, numbers and operations, attending to pupil explanations, and assessing pupil understandings more difficult. According to Delaney (2010), primary school teaching is work that makes high demands on teachers' mathematical knowledge.

Numeracy and educational disadvantage

The theme of sociological differentiation in educational attainment has held a prominent place in sociological research. Research studies consistently indicate that educational attainment reflects prior differences in social class background, parental education and/or income across national contexts and time periods (Shavit and Blossfield, 1993). In that context, it is acknowledged that children who do not learn to read, write, and communicate effectively are more likely to leave school early, be unemployed or in low-skilled jobs, to have poorer emotional and physical health, to have limited earning power, and more likely to be imprisoned (DES, 2011).

Poor numeracy skills create problems for people in everyday activities such as managing money or even helping children with homework. There are strong associations in the UK between poor numeracy skills and disadvantage and people with numeracy difficulties are more likely to be unemployed, to have social, emotional and behavioural difficulties, to be excluded from school and to be involved in crime.[16] In a world of technology the need for numeracy is becoming greater.

Bourdieu's work does not always explicitly specify the relative importance of different aspects of social and cultural capital, whether the main influence is parental education (and therefore parents' familiarity with the schooling system), the language and interaction style within the home, or the kinds of cultural and social activities in which middle-class families engage. Young people's engagement in high-culture activities with their parents is found to facilitate their educational progress and to account for at least some of the social gap achievement. The more recent longitudinal study GUI (ESRI, 2012) has reported similar findings. It must be stated however, that significant State-funded interventions – most notably the School Support Programme (SSP) under the auspices of the DEIS initiative - are in place in Ireland for a number of years in an effort to counteract the negative impact of socio-economic gaps on the educational achievement of our young people. The effectiveness of this investment remains to be seen, however.

Evaluation of practices in DEIS schools on numeracy and target-setting

An evaluation of aspects of practices in a representative number of DEIS schools has been published by the DES. According to the inspectorate's evaluation there were

[16] http://www.nationalnumeracy.org.uk/why-is-numeracy-important/index.html

positive practices generally evident among DEIS schools with regard to the implementation of numeracy strategies and interventions and there was significant improvement reported in pupils' progress in those schools (DES, 2011). However, in less than half of the schools evaluated significant pupil progress was not evident. Factors linked to the less than satisfactory progress made by pupils in those schools included weaknesses in target-setting practices such as inadequate analysis and use of assessment data in the target-setting process, unsatisfactory co-ordination of strategies and interventions as well as unclear teaching objectives. It was noted that the quality of target-setting practices used is particularly relevant to the subsequent progress made by pupils in their learning.

Planning processes in DEIS primary school

Numeracy is highlighted as a fundamental set of skills in the DEIS action plan and the inspectorate have reported on examples of effective practice with regard to numeracy in Effective Literacy and Numeracy Practices in DEIS schools (DES, 2009). The effectiveness of schools' planning processes to improve the numeracy skills of pupils, including the use of data to set targets and the monitoring of outcomes was evaluated by DES inspectorate (DES, 2011). The overall findings in relation to numeracy and the DEIS planning process were mixed. There was scope for development in the approach to target-setting used by more than half the schools. While good and very good work was evident in the majority of schools in relation to the strategies and interventions used to bring about improvements in the numeracy attainment levels of pupils, the fact that only 8 out of the 18 schools in this evaluation had so far succeeded in making such improvements in a significant way was described as being 'disappointing'. The evaluations carried out by the inspectorate suggest that the quality of the schools' target-setting practices in numeracy can have a bearing on the subsequent improvements in pupils' numeracy attainments. Accordingly, for a considerable number of schools, improving target-setting practices in numeracy should be a priority goal.

There is also a need for schools to reflect on the extent to which numeracy lessons and programmes are linked to the information the school has on its pupil performance and progress in numeracy. Fundamentally, according to this DEIS evaluation schools need to ensure that the numeracy targets set and the teaching approaches, interventions and strategies used in classrooms and other learning settings in the school are those that are most effective and appropriate for bringing about improvements in pupils' learning. Schools are required to analyse and take careful, practical account of assessment outcomes in setting numeracy targets. This requires numeracy assessment outcomes, target-setting and the work of teachers in classrooms and other learning settings in the school to be linked in a coherent and practical way so that improvements in numeracy levels can be achieved.

Strategies and interventions in DEIS schools

Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living. The majority of DEIS schools adopt programmes such as Maths Recovery, and Ready Set, Go Maths, which provide intensive and early intervention support to strengthen pupils' grasp of numeracy. Among the strategies used by many schools to improve numeracy standards were the following: team teaching, station teaching, increased emphasis in mental mathematics, differentiation of lesson objective setting for target pupils, incorporation of 'a problem a day' into the start of the lesson, standardisation of the mathematical language throughout the school, and a focus of life skills as they relate to numeracy (DES, 2009). Games, practical equipment, ICT, and commercial programmes to assist with particular aspects of numeracy were also in use.

Concluding Comment

This chapter briefly examined some of the assessments, studies and surveys, including an evaluation of DEIS, carried out in recent years as they pertained to Ireland. Many conclusions can be drawn from the national assessments and evaluations presented and discussed. The findings of national and international assessments of mathematics achievement at both primary and post-primary level indicate that Irish students find problem-solving and the areas of measures more difficult than other areas of the mathematics curriculum. The findings also indicate that fewer students in Ireland perform at the highest-achievement level in comparison to the OECD average. These findings have implications for curriculum development and for teacher education, both initial teacher education and teacher professional development.

4

The Language of Maths

A greater emphasis has been put on mental and oral maths in the primary school mathematics curriculum and it is recommended that mental/oral maths activities form an integral part of every mathematics lesson (Sharpen Brains Not Pencils, INTO Online Summer Course, 2011).

The skills of discussing, communicating, expressing, making connections, reasoning, hypothesising, explaining and recalling are fundamental in the teaching of mathematics. In the infant classes the focus of every maths lesson is on 'talk and discussion'. It is vital that an equal balance of oral and written maths is maintained through the middle and senior classes:

> Sadly, children are frequently expected to write mathematics before they have learned to imagine and discuss, and those who do not make the connection are offered more paper and pencil work instead of the vital talk and discussion. Yet in other subjects it would be unthinkable to ask children to write about what they cannot say.[17]
>
> (Anita Straker, 1993)

The significance of language and the ability to communicate are essential elements of teaching and learning in all curricular areas as they allow information to be transmitted between the teacher and the student. It must be acknowledged that mathematics has a unique language with its own symbols, vocabulary, grammar, semantics and literature. The current growth in electronic and mechanical devices may require us to do less computation 'in our head', although it is still essential that children acquire proficiency in arithmetical computation, it is now more important the child develops greater conceptual understanding of what is going on behind the processes and is able to apply them constructively in unfamiliar situations (Woodward, 2008; Deboys and Pitt, 1988). This understanding requires children to be able to represent mathematical ideas in a variety of ways and mathematical language allows us to connect the different representations and support 'abstract thinking'. Mathematical literacy allows children to develop the life skills of 'applying and problem-solving; communicating and expressing; integrating and connecting; reasoning; implementing, understanding and recalling' (Dooley and Corcoran, 2007, p.222).

The communication skills of turn-taking, listening to other opinions and expressing one's own opinion are essential elements across all areas of the curriculum but mathematics literacy also requires the learning and understanding of the terms and vocabulary and the interpretation of symbols that are particular to maths. The capability

[17] http://moodle.intolearning.ie/webdav/TEST/modules/intro/grasshopper_or_inchworm.html

to talk about mathematics is important for both teacher and learner as it facilitates the transmission of mathematical knowledge:

> As language permits mathematics learners to work out meanings, to convey their understanding, (to) develop their thinking, and to express their answers with others children need to be competent in both in the 'language of instruction' and in the 'language of mathematics (the mathematics register)' and it is essential that the teaching and learning of mathematics incorporates language aspects and should include listening and discussion (oral language) as well as reading and writing (written form).
>
> (Máire Ní Ríordáin, 2009)

What is the mathematics register? We must consider mathematical language as a distinct 'register' within but often different to the naturally spoken 'first language' of our pupils. One aspect of the mathematics register is the specific vocabulary and specialist terminology of math. Words like rectangle, denominator, per cent are 'maths words' and are rarely met outside mathematic literature or texts. Pupils may only encounter this vocabulary within a mathematics setting and therefore it is unlikely to be reinforced outside the confines of the mathematics lesson. As well as highly specialised mathematical language, symbols also feature regularly e.g. + - ≤ ≥ ÷ × €, and is one of the most distinctive features of mathematics. Students are often required to interpret text and convert it to symbolic representation or vice versa. They may also be required to perform operations using these symbols and engage in discussion on their findings. The register also has words and phrases 'borrowed from everyday language' which may have different meanings in the two contexts. This ambiguity needs to be addressed to avoid being a source of confusion (Pimm, 1987).

Examples of some borrowed words/ Ambiguous terms:

Above, as great as, average, base, below, between, circle, collection, common, complete, degree, difference, different, divide, down, element, even, expand, face, figure, form, grid, high, improper, integration, leaves, left, little, low, make, mean, model, natural, path, place, point, power, product, proper, property, radical, real, record, reflection, relation, remainder, right, root, row, same, sign, similar, square, table, times, top, union, up, value, volume

According to Durkin and Shire (1991), interpretation of this everyday language can further be hindered within certain contexts:

- Conditions (if, when)
- Comparatives (greater than, the most)
- Negatives (not, without)
- Inferentials (should, could, because, since)
- Low information pronouns (it, something)
- Lengthy passages where the structure and phrasing of some questions may provide inadequate 'language clues/context' to help with their interpretation

Carter and Quinnell (2012) describe some other lexical difficulties experienced by pupils:

- Cardinal numbers are adjectives when used in English (five boys) but are nouns when used mathematically (the answer is six). In the cardinal number system 2 is higher than 1 but in the ordinal number system, first is higher than second.
- Pronunciation of numbers thirteen to nineteen is similar to thirty, forty, etc.
- In fractions the numerator follows the pattern of cardinal numbers; the denominator, however, follows the pattern of ordinal numbers with the exception of halves and quarters. Sometimes fractions are described as five over six.
- In the case of decimals the cardinal system of number is used to describe the 'whole number' but the decimal part is described as single digits – 563.125 is described in words as five hundred and sixty three point one two five. The failure to verbalise decimals correctly may not be a misunderstanding of place value but a difficulty with the lexicon of number.
- Mathematical sentences might not conform to left to right orientation which is so important in English. Consider this numerical sentence and the order of operations which are needed to solve it: $1 + 2 (6 - 4) \div (3 \times 4)$

There is certainly a need to dedicate discrete time and specific teaching to ensure children acquire and understand the terminology of mathematics. Carefully guided instruction and modelling are crucial when introducing new vocabulary or concepts. Sutherland (2007, p. 130) stresses that the "teacher is the gateway to the world of mathematical language … the key agent for creating a mathematical micro-culture". The teacher's role is to be a guide, a model who provides rich learning opportunities for the students. To create a learning community and to optimise the levels of communication (oral language), group work is important: "as groups work with materials, pictures and context, translating between different modes, their discussion becomes a place to share and consolidate emerging ideas" (Cramer and Karnowski, 1995)

There are many ways in which teachers incorporate and facilitate the teaching of mathematical language in their classroom. The Ready Set Go Maths Programme, initially titled 'Success in Numeracy' was piloted between 1999 and 2001 following research by the Northern Ireland Steering Group for Numeracy to investigate factors which affect some children's level of achievement in numeracy and to identify approaches to bring about improvement. The programme places great emphasis on the use of manipulatives in young children's learning while stressing that this must go hand-in-hand with a well-planned use of language:

> Early in the project the importance of the children's use of language within mathematics became a priority, the children need to be 'engaged' in discussion in order 'to develop their ability to express their thinking clearly, to support those who are tentative in expressing themselves orally, and to assess the children's use of understanding.
>
> (Pitt. E. 2006, p.9)

During the piloting of the programme this emphasis was further heightened "with teachers increasingly aware of giving instructions clearly, of explaining ideas carefully and of posing questions to elicit specific responses" (Pitt, 2006, p. 9). In this learning environment the children are actively involved with manipulatives and context. They discuss their ideas with each other and with the teacher who can assess the level of understanding or misunderstanding as the case may be.

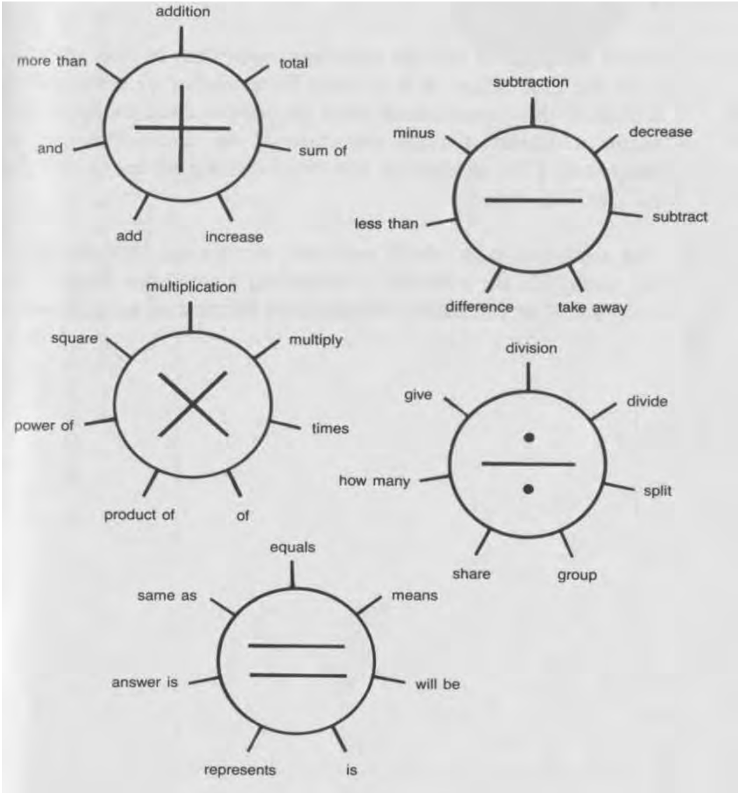
When the National Numeracy Strategy was published in England in 1999, it recommended a booklet on the "words and phrases that children need to understand and use if they are to make good progress in mathematics" (DfEE, 1999, p.1). It identified for teachers, mentors, support staff and parents appropriate mathematical language in four pages of vocabulary checklists ranging from vocabulary required for the five strands (number and the number system, calculations, solving problems, handling data and measures, shape and space) to commonly used instruction vocabulary from tests and texts. A structured approach to the teaching of maths language with associated 'real objects', mathematical apparatus and pictures and/or diagrams, as well as the informal everyday language of the maths class is advised. The importance of repetition and the inclusion of the vocabulary in oral sessions are stressed as methods for sorting ambiguities and misconceptions. The strategy document also contained some useful ideas on questioning which require the student to recall facts, apply facts, hypothesise and predict, design and compare procedures, interpret results and apply reasoning (DfES 12/00; DfEE, 1999, p.4).

In the Irish context, the Primary School Curriculum and Teacher Guidelines (DES, 1999) provide a limited list of symbols, numerals, fractions and terminology associated with each class (Teacher Guidelines, Mathematics, 1999, p. 70) and a short glossary of mathematical definitions for teachers (Teacher Guidelines, Mathematics, 1999, pp. 76-77). At professional development seminars held during 2012, the Professional Development Service for Teachers (PDST) suggested that each class teacher should formulate a list of synonyms for the different mathematical operations and pass it to the next teacher when the class was being promoted. The PDST suggested that it should be added to each year, providing a comprehensive list of the vocabulary that children encounter as they go through the primary school. The following list while not comprehensive, serves as an illustration. It has been compiled from a variety of sources.

Addition	**Subtraction**	**Multiplication**	**Division**
+	-	×	÷
Accumulate	Backwards	Array	Half, third,
Add	Decrease	By	Distribute
Altogether	Debit	Double, triple, etc.	Divide
And	Deduct	Twice, thrice, etc.	Divisible
Another	Difference	Square, cube, etc.	Diviser
Augment	Diminish	Two-fold, Three-	Factor
Bigger (than)	Discount	fold, etc.	Fraction
Credit	Down	Factor	Remainder
Grows	Exceed	Groups of	Group
Deposit	Fall	Lots of	Left
Extra	Fewer (than)	Magnify	Out of
Faster (than)	From	Multiple	Parts
Forward	Gone	Multiply	Per
Further (than)	Leave	Of	Per cent
Gain	Left (over)	Product	Portion
Greater (than)	Less (than)	Repeated	Quotient
Heavier (than)	Lighter (than)	Taxation	Rate
Higher (than)	Lose	Times	Reciprocal
Increase	Lower (than)		Remainder
Longer (than)	Off		Share
More (than)	Narrower (than)		Split
Older (than)	Nearer (than)		
Positive	Net income		
Plus	Minus		
Rise	Negative		
Sum	Reduce		
Taller (than)	Remaining		
Thicker (than)	Remove		
Together	Reverse		
Total	Thinner (than)		
Up	Shorter (than)		
Wider (than)	Slower (than)		
With	Subtract		
	Take away		
	Withdraw		
	Younger (than)		

This list can be used as a checklist to revise vocabulary and to reinforce the previous year's work.

Ann Henderson (1989, p. 29) suggests a simpler format of recording the language of operations that might be more suitable for Junior Classes or for children with specific learning difficulties.



The National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL), which is based in the University of Limerick, has some very interesting resource and research guides. Volume 1 (1) 2009 describes some practical activities and games that facilitate the teaching of mathematical language:

- ‘Just a Minute’ is a matching game of vocabulary and symbols;
- ‘Language of Mathematics’ is an activity based on the teacher providing flashcards of mathematical terminology which require the children to (1) make a statement in everyday language about the term, (2) give a symbolic/pictorial representation of the term; (3) make a mathematical statement using the term;
- ‘Generalisation’ is an activity where groups of children discuss some general statement together and decide on whether it is ‘always true’, ‘sometimes true’ or ‘false’ (e.g. if you multiply 11 by any number the answer will always be greater than 11);
- ‘Poster Presentation’ gives the children an opportunity to represent and communicate their understanding of a mathematical concept;
- ‘Mathematical Dictionary’ which allows children to keep a record of the vocabulary they encounter.

5

Problem-Solving in Mathematics

Introduction

The introduction to the mathematics curriculum emphasises that while mathematics is an “essential tool for the child and adult”, it is also an “intellectual pursuit in its own right, a source of fascination, challenge and enjoyment” (DES, 1999, p.3). These two key stances from which mathematics can be viewed are not by any means mutually exclusive, yet it is worthwhile to bear them in mind, along with other considerations, as we discuss the whole area of problem-solving in the context of Mathematics education.

In the opening paragraph of the Primary School Curriculum: Mathematics (DES, 1999), mathematics is described as involving “the handling of information, the making of predictions and the solving of problems through the use of a language that is both concise and accurate” (DES, 1999, p.2). The curriculum goes on to say that mathematics education “gives the child a language and a system though which he/she may analyse, describe and explain a wide range of experiences, make predictions, and solve problems” (p.2). Thus the centrality of problem -solving in mathematics is emphasised from the outset.

The following paragraph on problem-solving is worthy of particular focus at this point, since it provides a clearer picture of the context within which problem-solving is set in our curriculum:

> Developing the ability to solve problems is an important factor in the study of mathematics. Problem-solving also provides a context in which concepts and skills can be learned and in which discussion and co-operative working may be practised. Moreover, problem-solving is a major means of developing higher-order thinking skills. These include the ability to analyse mathematical situations; to plan, monitor and evaluate solutions; to apply strategies; and to demonstrate creativity and self-reliance in using mathematics. Success helps the child to develop confidence in his/her mathematical ability and encourages curiosity and perseverance. Solving problems based on the environment of the child can highlight the uses of mathematics in a constructive and enjoyable way.
>
> (DES, 1999, p.8)

Prior to directing attention to the key strand areas of the mathematics curriculum, six broad objectives for skills development across the mathematics curriculum are detailed, the first of them relating to problem-solving skills: “the mathematics curriculum should enable the child to ... apply mathematical concepts and processes, and plan and implement solutions to problems, in a variety of contexts” (p.12). Problem-solving is a skill that permeates the various strands of the curriculum, and a skill that is given pride of place therein. This pride of place is mirrored in the opening section of the curriculum sections for each class grouping, where more specific detail is given regarding the aspects of problem-solving to receive particular attention in the various classes. The Teacher Guidelines that form part of the Primary School Curriculum state that mathematical problems include:

- Word problems
- Practical tasks
- Open-ended investigation
- Puzzles
- Games
- Projects
- Mathematical trails

(DES, 1999, Mathematics – Teacher Guidelines, p.35)

It is interesting to note that many of the textbooks that are in use since the Primary School Curriculum was launched in 1999 contain a predominance of word problems in dedicated sections in the various chapters, with less evidence of a problem-solving approach permeating the overall structure of the chapters that one might have anticipated from the stance on problem-solving that is espoused in the Curriculum handbooks. Of course, the textbook is just one tool in the overall mathematical experience of the child. Nevertheless, it prompts one to examine our whole understanding of what problem-solving is all about, and how we can or how we might foster skills in this area in the children with whom we work. Accordingly, it is appropriate at this juncture to visit some of the literature around problem-solving in the Mathematical area and consider how it matches our thinking and practice in Mathematics teaching and learning.

Problem-solving and *Aistear*

Prior to referencing the literature, however, it is appropriate to probe Aistear's perspective on problem-solving, as a child's early mathematical experience and experience of problem-solving will underpin future development. Aistear, the Early Childhood Curriculum Framework (NCCA, 2009) for all children from birth to six years in Ireland, points out that early learning is not just important in its own right but that it also lays important foundations for later learning. Aistear contains numerous references to problem-solving - it does not confine problem-solving to a discrete area but rather presents it as being a facet of learning across a range of areas. Aistear is based on 12 principles of early learning and development. These principles include, for example, active learning, play and hands-on experiences, relevant and meaningful experiences, and communication and language. Aistear has at its core children learning by doing things i.e. active learning. It speaks of children using their senses to explore and work with the objects and materials around them. Through these experiences, and through interacting with the children and adults around them, the children are seen to develop the dispositions, skills, knowledge and understanding, attitudes and values that will help them to grow as confident and competent learners. Interweaving problem-solving opportunities with these experiences can serve to not only foster problem-solving skills at that point in time but can also foster positive attitudes to problem-solving in children's future learning and in their lives in general.

For example, in Aistear's Wellbeing Theme, sample learning opportunities include encouraging toddlers to solve problems and to think about different ways of doing things. Interestingly enough from an attitudinal perspective, it highlights also the importance of acknowledging and encouraging toddlers' problem-solving efforts. Similarly, the section regarding the Wellbeing Theme and young children stresses the importance of the adult using a problem-solving approach to respond to social conflicts, and the importance of encouraging children to restate a problem, helping them to find a solution, providing subsequent support and ensuring all involved are supported, comforted and reassured. There is a strong focus on the child being meaningfully and actively involved in engaging with a problem – the potential for transferring such meaningful engagement to a mathematical context should not be underestimated.

Problem solving in Mathematics – what is this understood to mean in our classrooms?

Before considering children's learning around problem-solving, it is worthwhile to consider some definitions of problem solving in the literature. Schoenfeld (1992, p.10) points out that 'problems' and 'problem solving' have had "multiple and often contradictory meanings through the years – a fact that makes interpretation of the literature difficult". He goes on to refer to two poles of meaning that are included in Webster's definitions for the term 'problem':

- In mathematics, anything required to be done, or requiring the doing of something, and
- A question…that is perplexing or difficult

(Webster, 1979, p.1434)

The glossary that is included in Aistear (NCCA, 2009, p.57) includes the following explanation for problem-solving: "Problem-solving refers to children's ability to overcome obstacles that they meet while playing and undertaking activities". A further interpretation presents problem-solving as "engaging in a task for which the solution is not known in advance"[18]. There are indeed numerous definitions of problem-solving, but a common strand running through the vast range of these definitions is the notion of challenge or extra focus that is inherent in problem solving over and above that which is associated with more routine tasks. The degree of challenge can be quite varied, however, ranging from processes which border on the automatic to significant challenges to one's thinking. Bagnall makes the following interesting observation:

> Looking at the whole issue of problem-solving I find that there are many views as to what it really means. It seems to go from something that is not just a written calculation such as, '5 + 7 = ?', but instead words are used, 'Jane had 5 pencils and her brother gave her 7, how many did she now have?', to something far more complicated in which the problem is multifaceted: 'Design a system for traffic lights to operate in, when there is a junction of three major roads and …[19]

He suggests that problem solving may be regarded as "something that a pupil becomes involved in when s/he has to do some extra exploration in order to come up with a solution". Even this limited perusal of various definitions/interpretations of problem-solving is sufficient to prompt questions about traditional mathematics textbooks that may include 'problem-solving' sections in each chapter, often containing a preponderance of similar tasks that may become rather repetitive in nature once the first question has been 'cracked'. While it may indeed be of value to provide practice to pupils in solving problems of a similar nature, if problems are routinely presented as repetition of a formula applied to a series of similar questions, to the exclusion of non-uniform problems or challenges, then children may not be gaining sufficient opportunities to develop their problem-solving skills. Bagnall makes a further point that more able children may produce an instant solution to problems, therefore, that it is

[18] http://www.nctm.org/standards/content.aspx?id=26860.

[19] http://nrich.maths.org/7231

important to enable less able children to engage in problem-solving activities. This underlines the importance of differentiation when creating problem-solving opportunities in our mathematics lessons and beyond. Problem-solving activities that are too challenging for some or all pupils lose their value since some pupils may not be in a position to engage with such activities at all.

These broad understandings of problem-solving have significant implications for primary school teachers, challenging teachers to ponder their expectations of problem-solving activities in the mathematics class and beyond. Teachers must ask themselves whether the tasks they set their pupils genuinely engage them in problem-solving, whether there are opportunities for all children, and if not, how they might address these challenges. If the problems presented are insufficiently challenging for some children, it is conceivable that those children may not be afforded opportunities to develop their problem-solving skills.

It is appropriate at this point to revisit Schoenfeld's reference to two of the definitions for problem-solving cited in Webster's Dictionary. According to Schoenfeld:

> Webster's Definition 1 … captures the sense of the term problem as it has traditionally been used in mathematics instruction. For nearly as long as we have written records of mathematics, sets of mathematics tasks have been with us -- as vehicles of instruction, as means of practice, and as yardsticks for the acquisition of mathematical skills. Often such collections of tasks are anything but problems in the sense of the second definition. They are, rather, routine exercises organized to provide practice on a particular mathematical technique that, typically, has just been demonstrated to the student
>
> (Schoenfeld, 1992, p.11).

A brief perusal of a selection of the mathematics textbooks used in Irish primary schools in recent decades suggests a strong leaning towards problems that align rather closely with such routine exercises that provide practice on mathematical techniques that have just been dealt with in previous sections of the chapter in question. The consequences of instruction based largely or even exclusively on such 'problem sets' may have a limiting effect on the possibility of developing true problem-solving skills in pupils. This may call into question the wisdom of heavy reliance on such textbooks to 'deliver' the requisite opportunities to pupils to develop their problem-solving skills. Of course, standardised assessment tests might also warrant scrutiny in this regard – do those that are currently available actually test the broad range of problem-solving skills that the Primary Curriculum emphasises? And, if not, do standardised assessment tests in their current form lend themselves to assessing such problem-solving skills? The deliberations on these questions are significant on a number of fronts – if such tests focus on only a limited range of problem types and problem-solving skills, might this lead to other problem-solving skills that do not lend themselves to being assessed so readily being afforded less importance in our classrooms? Schoenfeld (1992) argues that the consequences of instruction based largely or even exclusively on a narrow range of 'problem sets' may have a limiting effect on the possibility of developing true problem-solving skills in pupils- this may have implications for the Irish context.

Methods of Teaching and Learning

The NCCA stated that "the ability to reason mathematically and problem-solve is an essential skill for living in the 21st century"[20]. Accordingly, it is important that the learning opportunities that are generated in the primary school facilitate the development of this skill. Problem-solving skills, while highlighted in the Mathematics curriculum, have the potential for much transfer value to other subject areas and to a variety of life situations. The corollary of this is that teaching and learning in other subject areas and in non-subject specific aspects of school life may well have the potential to have a positive impact on children's mathematical problem-solving abilities.

Schoenfeld (1992) argues that students develop their sense of mathematics - and thus how they use mathematics - from their experiences with mathematics which occur largely in the classroom. He asserts, therefore, that "it follows that classroom mathematics must mirror this sense of mathematics as a sense-making activity, if students are to come to understand and use mathematics in meaningful ways" (1992, p.18). Again, this viewpoint has implications for how teachers approach problem-solving in schools – if teaching mathematics is purely a textbook-based activity, the question must be posed whether children are getting an opportunity to engage fully with real problems. If children are presented with a page of mathematical tasks that mirror a similar formula from one to the next, are they engaging in problem-solving in a real sense? Of course, in terms of the child's learning, there may be considerable merit in presenting opportunities to consolidate learning by repeated exposure to similar types of problem. However, the learning objectives must be clear – is the activity about affording opportunities to grapple with real problems or about allowing for practice and consolidation? Other points to bear in mind may include the time factor – problems in life do not always neatly fit into five or ten-minute slots – some problems may warrant re-visiting over a period of days or longer before workable solutions are arrived at or some problems may have multiple solutions or multiple paths to one solution. An overly formulaic approach to problem-solving may narrow the potential for development of thinking skills and may have a stifling effect on children's problem-solving approaches. Similarly, an overly textbook-orientated approach could conceivably limit the pupils' sense of identification with the problem and hence their interest in solving same. Linking problems to real-life situations or bringing real-life problems to the classroom may result in the children having a vested interest in solving the problem and may do much to enhance their engagement with same.

The NCCA reports that its research shows teachers' strong ownership of the child-centred theories underpinning the Primary School Curriculum in contrast with their limited ownership of child-centred teaching and learning methods[21]. NCCA findings showed that teachers needed much greater exemplification of methods of teaching and learning with the Primary School Curriculum including:

- Active learning
- Environment-based learning

[20] http://www.ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/Information_on_Project_Maths/Parents_info_note.pdf - PAGE 1 – accessed 2nd January 2013

[21] http://www.ncca.ie/en/Curriculum_and_Assessment/Early_Childhood_and_Primary_Education/Primary_School_Curriculum/Primary_Curriculum_Review_PCR_/Phase_1_English,_Visual_Arts_and_Mathematics/#3

- Collaborative learning
- Differentiated learning (particularly in multi-class settings)
- Higher-order thinking and problem solving.

The NCCA is currently developing a website that will show what teaching and learning with the curriculum looks like in different class contexts. ACTION (Assessment and Curriculum Teaching Innovation On the Net) will provide a platform for showcasing different teaching methodologies that can be used with the Primary School Curriculum. Internet video, pod casts, samples of children's work, samples of teachers' materials, photographs and various other support materials will be housed on ACTION at www.action.ncca.ie).

Teachers' dispositions towards methodologies, as referenced by the NCCA above, are relevant to the thinking around the teaching and learning of problem-solving in our schools. It is strongly arguable that teaching and learning methodologies have a significant impact on the learning that takes place and on pupils' overall attitudes to mathematics in both the short and long term. Schoenfeld (1992) contends that instruction has traditionally focused on the content aspect of knowledge. In other words, traditionally one defines what students ought to know in terms of subject matter, and characterizes what a student knows in terms of the amount of content that has been 'mastered'. Schoenfeld cautions that "as natural and innocuous as this view of 'knowledge as substance' may seem, it has serious entailments" (1992, p.25). From such a perspective, 'learning mathematics' is defined as mastering, in some coherent order, the set of facts and procedures that comprise the body of mathematics. While this has obvious merit, such a route to learning may not enable the pupil to sufficiently gain practice in the area of problem-solving.

The NCCA[22] draws our attention to the fact that the "Primary School Curriculum emphasises real-life problem-solving, using checkable answers and activities based in the student's environment. It points out that "encouraging students to engage with open-ended problems such as making scarves for teddy, building a house using a limited number of bricks, or working out how to spend a sum of money on food for a party will help them to realise that there are many ways to solve a problem, and that sometimes there is more than one 'right' answer". Such a focus on problems that are anchored in the reality of the child's daily existence may do much to engage a child in purposeful and focussed thinking that is centred on meaningful problem-solving.

Piggott and Pumfrey[23] (2007) posit the idea that when we are presented with a mathematical problem, it is only a problem if we do not know immediately how to solve it. They compare the process of problem solving to a journey from a state of not knowing what to do, towards a destination which we hope will be the solution. The key is to have some strategies at our fingertips which will help us to identify a possible route through to a solution. This approach underlines the importance of the role of the teacher in nurturing the child's problem-solving skills and in creating the right environment for children of varying abilities to engage fully with the problem-solving process. A mathematical journey is often full of twists and turns where ideas are revisited or where one needs to step back and look for alternatives. It is no mean feat for a teacher to cater for the broad spectrum of pupil ability – ranging perhaps from the exceptionally able to children with learning disabilities of varying degrees – such that they will have their interest hooked by the problems presented and will be sufficiently, yet not excessively,

[22] http://www.ncca.ie/uploadedfiles/P_Mild_Maths.pdf
[23] *http://nrich.maths.org/5456/index*

challenged. The teacher will need to be mindful, not alone of the children's prior learning in mathematics but also of a host of other factors such as their general ability, their attitude to problem-solving and mathematics in general, and their overall disposition in terms of persevering in the face of challenge. For some children, a mistake or dead-end may give vital clues to the mathematics of the problem and therefore, may be crucial in the solution process – for others it may dampen their enthusiasm. All of these dimensions need to be taken into account by the teacher when pitching problems for the target group.

Bagnall[24] sees the behaviour of the teacher as pivotal in the advancement of children's problem-solving capabilities. He cautions against asking a question in such a way that it might appear to the children that the answer was in the teacher's head or in speaking in such a way so as to bring about a state of anxiety or having to rush in order to get an answer. All of this reflects real-life problem-solving – a mutual endeavour to respond to a real challenge rather than second-guessing a solution at which a person may have already arrived. Bagnall feels that such a stance helps in allowing pupils to realise that their thoughts are valued and encourages children to think for themselves, and not just remember what the teacher said on previous days. In short, Bagnall sees the teacher as the creator of the environment that allows problem-solving to take place. It is arguable that an approach such as this may additionally have long-term benefits in terms of fostering in the child a positive attitude to mathematics – while such benefits may not be easily measurable in the short term, they may nonetheless be of significant value to the child as an individual and to society as a whole in future years.

Piggott[25] indicates that she often starts a problem-solving session by saying something like:

> Today we are going to be problem-solving and this means working on problems. So, before we start it might be good to know what I mean by a problem. A problem is something that you probably will not immediately see how to tackle or which once started will challenge your thinking. This means that at some point you are likely to get stuck and this is OK. In fact it is important that you get stuck so that we can talk about possible ways of getting unstuck.

She explains that the very problem with problems, namely that they should result in people being stuck, is at the heart of what problem-solving is about. Such a scenario contrasts strongly with most classroom experiences of learners when being stuck is equated with failure. Taking cognisance of this in presentation of problems and in scaffolding pupil learning around same is likely to have a significant impact not only on the development of children's skill in the area of problem solving but ultimately in their attitude to the whole area of problem solving and the consequent energies that they will bring to this work.

Much attention has been drawn in the literature to the physical organisation of children in classes in terms of how this may impact on their engagement with problem-solving. There are many questions that teachers might ask themselves. For example, are children in groups during problem solving? Working individually? Working collectively? Or in combinations of these? Are problems set for all the class, or have children opportunities to direct themselves to some degree on problems that are of particular interest to them or that are specifically matched to their perceived areas of strength or difficulty?

[24] http://nrich.maths.org/7231
[25] http://nrich.maths.org/6294

Reynolds and Muijs (1999) state that the advantages of co-operative small group work to developing problem-solving skills lie partly in the 'scaffolding' process whereby pupils help each other learn in the 'zone of proximal development'. The merits of addressing mathematical problems in groups are numerous. Reynolds and Muijs point out that giving and receiving help and explanation may develop children's thinking skills, as well as helping them to verbalise and structure their thoughts. They cite the research of Peterson (1988) and Leikin and Zaslavsky (1997) in this regard. They also note that by cooperating and sharing in small groups children can share their own ways of thinking and reflect on them and on the thinking and ideas of others. It is possible that this may encourage pupils to engage in more higher-order thinking (Becker and Selter, 1996, cited by Reynolds and Muijs, 1999). Reynolds and Muijs also suggest that group work may help pupils who are less able problem solvers to overcome their insecurity about problem solving since in a group situation they can see more able peers struggling over difficult problems. They also point out that the synergy of the group has the potential to add significantly to the value of a group approach – the fact that a group contains more knowledge than an individual means that problem-solving strategies can be more powerful than those arrived at by individuals.

It is, nevertheless, necessary to draw attention to possible problems with group methodologies – Reynolds and Muijs (1999) highlight drawbacks such as the following: shared student misconceptions can be reinforced by group work; pupils can be tempted to engage in off-task social interaction; or pupils who feel they have little to contribute may become passive, leaving others to do all the work. While all teaching requires preparation by the teacher, for it to be effective, small group work requires a substantial degree of planning and preparation and well-honed classroom management skills. Nevertheless, programmes that have attempted to develop problem-solving skills through small-group work report good results, such as enhanced conceptual understanding and higher achievement on problem-solving tasks (Maher, 1991; Verschaffel and De Corte, 1993; Goods and Galbraith, 1996; Leikin and Zaslavaky, 1997; Townsend and Hicks, 1997; Wood and Sellers, 1997 – cited by Reynolds and Muijs, 1999, p.282).

In tandem with the above, Mercer and Sams (2006) speak of the claims that working and talking with partners while engaged in mathematics is beneficial to students' learning and the development of their mathematical understanding. And again, in line with Reynolds and Muijs, they are cognisant of the drawbacks of group-based problem solving in the classroom. They acknowledge that observational research has shown that primary-school children often do not work productively in group-based classroom activities, with the implication that they lack the necessary skills to manage their joint activity. They describe research that investigated these issues and also explored the role of the teacher in guiding the development of children's social skills in using language as a tool for reasoning. This research involved an interventional teaching programme called 'Thinking Together', designed to enable children to talk and reason together effectively. The results obtained indicate that children can be enabled to use talk more effectively as a tool for reasoning, and that talk-based group activities can help the development of individuals' mathematical reasoning, understanding and problem-solving. The results of this research also encourage the view that the teacher of mathematics can play an important role in the development of children's awareness and use of language as a tool for reasoning.

Mercer and Sams (2006) assert that, in their lives out of school, many children may rarely encounter examples of the types of discussion that are necessary for effective group problem-solving. Citing Mercer, (p. 510), they indicate that research has shown that teachers rarely make their own expectations or criteria for effective discussion explicit to children (Mercer, 1995). Their argument is worthy of particular note, in so far as it challenges teachers to do much preparatory work around creating the building blocks for effective group work in its own right before expecting children to derive optimum benefit from applying a group approach to problem-solving:

> Children are rarely offered guidance or training in how to communicate effectively in groups. Even when the aim of talk is made explicit – 'Talk together to decide'; 'Discuss this in your groups' – there may be no real understanding of how to talk together or for what purpose. Children cannot be expected to bring to a task a well-developed capacity for reasoned dialogue. This is especially true for the kinds of skills which are important for learning and practising mathematics, such as constructing reasoned arguments and critically examining competing explanations.
>
> (Mercer and Sams, 2006, p .510)

On this basis, it would appear logical that children need not only guidance in the relevant knowledge of mathematics itself but also in learning how to use language to work effectively together. Mercer and Sams found that that providing children with guidance and practice in how to use language for reasoning enables them to use language more effectively as a tool for working on maths problems together. This point has implications for primary teachers. To derive maximum benefit from the methodologies employed, it is imperative that teachers have an awareness of the demands that these methodologies can make on any or all of the children in the target group. Equipping or not equipping the pupils with the requisite skills to engage fully with a particular methodology is likely to have a profound effect on the success or lack thereof of a given approach.

Effective Teaching of Numeracy

What does research tell us about highly effective teachers of numeracy? According to Hargreaves:

> Teachers don't merely deliver the curriculum. They develop, define it and reinterpret it too. It is what teachers think, what teachers believe, and what teachers do at the level of the classroom that ultimately shapes the kind of learning that young people get.
>
> (Hargreaves, 1994, p. ix)

In a major study of effective primary school mathematics teaching in the United Kingdom, Askew, Brown, Rhodes, Johnson and Wiliam (1997) studied the practices of a number of different teachers, with varying levels of effectiveness. Using mean class gains on a test involving aspects of the number system, computation, and problem solving, aurally administered by teachers with children writing their answers, teachers were grouped into broad categories according to effectiveness. Insights into the practices of effective teachers came primarily from focus schools and case studies of individual teachers, six of whom were identified as highly effective. The highly effective teachers

- encouraged students to describe their methods and their reasoning, and used these descriptions as a way of developing understanding through establishing and emphasising connections
- emphasised the importance of using whatever mental, written or electronic methods are most efficient for the problem at hand
- particularly emphasised the development of mental skills, and

- connected different ideas of mathematics and different representations of each idea by means of a variety of words, symbols and diagrams

Brown, Askew, Baker, Denvir, and Millett (1998, p. 373) noted that international observational studies ‘seem to show some agreement on some of the aspects of teacher quality which correlate with attainment’. These included the use of higher order questions, statements and tasks which require thought rather than practice.

According to the Early Numeracy Research Project in Victoria, Australia[26], effective teachers of numeracy have a number of characteristics. Effective teachers of numeracy have a mathematical focus, which they make clear to their pupils. They focus on mathematical ideas and structure purposeful tasks that engage pupils. They use a range of materials and contexts to represent concepts, use teachable moments as they occur and make connections with previous learning experiences. Effective teachers of numeracy use a combination of whole class, group and individual teaching. They probe and challenge their pupils’ thinking and reasoning through questioning and interaction, and encourage pupils to explain their mathematical thinking. Teachers build on pupil’s mathematical ideas and strategies and have high but realistic expectations of all children. They use a variety of assessment methods to inform their teaching. Effective teachers of numeracy are also confident in their own knowledge of mathematics and present mathematics as enjoyable.

There are a number of strategies that teachers can use in teaching pupils to solve problems. These are described by Delaney[27] as follows:

RUDE
• Read the problem
• Underline the key words
• Draw a diagram of the problem
• Estimate your answer and then solve the problem

STAR
• Search the word problem (info)
• Translate the words into an equation or picture (plan)
• Answer the problem (solve)
• Review the solution (check

LUV2C
• Look
• Underline
• Visualise
• Choose Numbers
• Calculate

Delaney argues that a good problem should leave the solver feeling ‘stuck’ at first, relates to the children’s experience, connects different maths topics, allows children with different attainment levels to achieve success with it, may take time to complete and requires children to justify and explain their answers and methods.

---

[26] http://www.education.vic.gov.au/Documents/school/teachers/teachingresources/discipline/maths/enrpreport.pdf

[27] Sean Delaney, http://seandelaney.com/presentations/

Concluding Comment

Problem-solving is a key element of all mathematics learning. In everyday life, being reasonably adept at solving problems can be very advantageous. Exposure to meaningful problem-solving and to problem-solving strategies should be embedded across the curriculum – a disposition to active engagement with problem-solving needs to be nurtured from an early age. The following comments from the National Council for Teachers of Mathematics[28] provide food for thought:

> Teachers play an important role in developing students' problem-solving dispositions. They must choose problems that engage students. They need to create an environment that encourages students to explore, take risks, share failures and successes, and question one another. In such supportive environments, students develop the confidence they need to explore problems and the ability to make adjustments in their problem-solving strategies.

Teachers, therefore, need to be supported through continuing professional development to enhance their own knowledge and skills in mathematics teaching in order to optimise their pupils' learning.

[28] http://www.nctm.org/standards/content.aspx?id=26860

6

Teaching all Children Mathematics

Introduction
Peter Westwood, in his book on common sense teaching strategies for children with special needs cites Lerner and Kline's research (2006) which indicated that as many as 6% to 7% of children with and without disabilities or learning difficulties experience major problems learning maths. A much higher number, however, are observed to be 'low achievers in mathematics and displaying a poor attitude to the subject and having no confidence in their own ability to improve' (Westwood, 2006, p.177-179).

In recent years the term 'mainstream education' cannot be separated from 'special needs education', 'education for disadvantaged children', 'multicultural education'. The range of abilities and the number of children from a wide spectrum of socioeconomic circumstances and cultural backgrounds that present in the classroom today, challenge teachers to address all their needs requires the teachers to question their assumptions about the teaching of mathematics and the mathematics curriculum:

> Our concept of mathematics as a rule-driven, computation-dominated curriculum actually creates much of the difficulty we have in coping with the range of children's talents and abilities. . What is critically important, however, is the ability to reason and solve problems.
>
> Van de Walle, 2004, p. 92)

Differentiation is now a buzz word in teacher planning. Westwood (2007) identifies with Howard Gardner's (1991) Entry Points and argues that differentiation should be directed to providing alternative pathways to enable all children to study the same curriculum content and achieve satisfactory outcomes rather than changing the curricular content. Sometimes some extra additional individual help will be required but this can frequently be given in the mainstream classroom using a cooperative teaching model, peer tutoring and group work, combining the talents of a class teacher and a support teacher to benefit all the differing needs in the class. Westwood's concern that 'watering down the curriculum ... can have the long term effect of increasing the achievement gap' between students with learning difficulties and their peers, 'exaggerating the effect of individual differences' and 'perpetuating inequalities among students' (Westwood, 2007, p. 200).

The revised Mathematics Curriculum of 1999 wholeheartedly encompasses this philosophy. Its opening lines describe the need to foster 'creative and aesthetic development', to enhance 'the growth of reasoning' through investigation and the development of 'confidence in effective communication' (DES, 1999, p.2) The design of the curriculum requires that that each class programme should be an 'interesting and meaningful experience' and should be 'sufficiently flexible' to accommodate and reflect the needs of all children in the class (p.3). Individual differences in ability, attainment, interests and learning styles require the teacher to understand how the conceptual knowledge in mathematics is developed and to ground their teaching in 'real contexts and situations'.

> The whole purpose of mathematics education is to acquire information, skills and strategies that enable an individual to solve problems … Children need to be taught how and when to use computational skills and problem-solving strategies for authentic purposes.
>
> (Westwood, 2007, p.190)

General Learning Difficulties

The Learning Support Guidelines (2000) advise that the child's conceptual development is best fostered with a gradual movement from working with concrete materials to the semi-concrete and eventually to the abstract when he/she is ready (DES, 2000, p.85). The Teacher Guidelines for mathematics (1999) note that the content of a good mathematics programme is sequential (DES, 1999, p.25) while the Curriculum (1999) document emphasises that children acquire an understanding of mathematical ideas in an 'uneven and individual way' (DES, 1999), p.5). So that we can build on the already acquired knowledge of the student, the issue of teachers' preparedness is therefore crucial and involves a combination of assessment, planning and teaching methodologies. The assessment stage, in particular, needs to focus on the child's abilities and learning styles as well as existing knowledge and avoid pressure to 'push the child to premature mechanical mastery of computational facts and procedures' (DES, 1999, p.5). While admitting that some children with limited abilities need more time to absorb information, Westwood challenges the traditional notion that pupils with learning difficulties are placed in 'low ability' groups and given 'a watered-down version of the mainstream curriculum' (Westwood, 2007, p.180). This approach may be called 'functional' or 'real-life' maths. The National Numeracy Strategy Framework in Britain states:

> The needs of the pupils regarded as 'special' are not essentially different from those of other children. Instead of focusing on differences, you might emphasise the links with the needs of all learners, and use them productively to improve learning opportunities for all children.
>
> (DfEE 1999, p.21)

The predominant model for instructing and supporting learning-disabled children has viewed the child as passive, with the mastery of skills being predominant over understanding, and ensuing placement or 'pulling-out' of these children to the 'special room'. Van de Walle cites work by Thornton and Bley (1994) as follows:

> Nearly every author in NCTM's excellent book Windows of Opportunity: Mathematics for Students with Special Needs leaves the reader with the same conclusion. Put simply, there is no need to change the content of the curriculum for students with learning disabilities. What must be done is what all good teachers do - pay careful attention to the child and how he/she learns and design instruction (not content) that maximises the strengths of the child while minimising the impact of weaknesses.
>
> (Van de Walle, 2004, p.93)

Specific Learning Disabilities and Dyscalculia

Specific Learning Difficulties (SLD) are often identified by a psychological assessment and the earlier this diagnosis is made the better. The specific difficulty may be a perceptual difficulty, either visual or auditory, a memory deficit (long and/or short term) or difficulties with attention span, impulse control or ADD/ADHD. All of these difficulties will affect the learning and therefore, the teaching methodologies of mathematics. None of them, however, should preclude the involvement of the child in any of the strands of the mathematics curriculum.

Butterworth and Yeo (2004, p.1) describe dyscalculia as a 'specific deficit of a very basic capacity for understanding numbers which leads to a range of difficulties in learning about number and arithmetic'. Anne Neville's research published in 2012 recommends early intervention as being of the 'utmost importance ... due to the cumulative nature of the mathematics curriculum' (p.13) and the deficits of basic number skills associated with dyscalculia. While withdrawal support in the junior classes is recommended the importance of common strategies and targets both in the classroom and Learning Support / Resource (LS/R) setting is also stressed. In-service training in co-teaching, differentiation and peer tutoring are also advised to assist teachers in providing for differing needs within the class. Neville's research indicates that foundation numeracy teaching is frequent in the LS/R class for senior students and therefore many of the children are not allowed to access 'the cumulative mathematics curriculum whilst also being denied opportunities within the classroom to practice and overlearn foundation skills ... which foster competence and confidence' (Neville, 2012, p. 12). She cites Dowker (2004, p.33) who implies that group activities structured at varying levels of complexity would suit varying and individual learning needs. These activities should build on the child's current knowledge and could be implemented through co-teaching and/or peer tutoring as recommended by DES (DES 1999; DES, 2009). The 'keep up' with the class curriculum approach which Neville's research shows as prevalent, may lead to increased withdrawal-type support and a separate classroom programme in senior classes (Neville, 2012, p.12).

Very often children with Specific Learning Difficulties are capable children with significant strengths. Identification of these strengths using diagnostic tests is crucial. Once identified, the teacher needs to adapt his/her teaching style and 'adapt instructional strategies to avoid weaknesses and capitalise on strengths without major modification of the curriculum' (Van de Walle, 2004, p.93).

Gifted and Talented

There is a growing awareness in Ireland of the needs of gifted children and primary school teachers' recognition of 'giftedness' is fundamental. But what is a gifted child?
The Draft Guidelines on exceptionally able students published by NCCA explain:

> There is no universally agreed term for students who generally would be described as 'exceptionally able'. Some descriptions include genius, gifted, very bright, high flyer, very able and talented. The term exceptionally able is used … to describe students who require opportunities for enrichment and extension that go beyond those provided for the general cohort of students.
>
> (NCCA, 2006, p.9)

The USA Department of Education[29] defines gifted and talented students as follows:

[29] http://www.education.com/reference/article/gifted-creative-talented-children/

Children and youth with outstanding talent perform or show the potential for performing at remarkably high levels of accomplishment when compared to others of their age, experience or environment. These children and youth exhibit high performance capabilities in intellectual, creative, and/or artistic areas, possess an unusual leadership capacity, or excel in specific academic fields. They require services or activities not ordinarily provided in the schools. Outstanding talents are present in children and youth from all cultural groups, across all economic strata and in all areas of human endeavour.

(US Department of Education, 1993, p. 3)

The NCCA guidelines suggest that approximately 5-10% of the school population may be exceptionally able and will demonstrate very high levels of attainment in one or more of the following areas:

- general intellectual ability or talent
- specific academic aptitude or talent
- visual and performing arts and sports
- leadership ability
- creative and productive thinking
- mechanical ingenuity
- special abilities in empathy, understanding and negotiation.

Within this group we could expect to find a minority of students (possibly 0.5%), who are profoundly exceptionally able with IQ range of 170+, often several years ahead of what is normally expected of their age group. There is no quantifying of the numbers who are exceptionally able at mathematics. According to Shiel et al (2010), however, there is concern that high-ability students are not being sufficiently challenged at present, as evidenced by their performance in the OECD Pisa International study (2009). Ireland has significantly fewer students at the higher proficiency levels (6.7%) in comparison to the OECD average (12.7%), the United Kingdom (9.8%) and Poland (10.4%). There are also proportionately more high-achieving students in Northern Ireland (10.3%) than in the rest of Ireland and in Finland (21.6%). Shiel et al (2010) argue that the pattern of results suggests that Ireland's low average performance is in part attributable to the comparatively low performance of the higher-achieving students.

Identification is the first step to fostering gifted pupils. The NCCA guidelines suggest that the following might be some of the characteristics of logical-mathematical (scientific) intelligence:

- can group and construct complex sets easily
- internalises and manipulates mathematical or scientific concepts
- can hypothesise and infer consequences
- can manipulate symbols
- grasps the steps in reasoning
- appreciates and creates patterns
- can see the solution intuitively

Some questionable practices continue to exist. Teachers often 'reward' highly capable students for finishing routine tasks quickly by having them do more exercises or more tedious exercises than required by the rest of the class. There is little value in this pursuit. Sometimes an 'enrichment' programme might be designed (puzzles or computer games) and while this succeeds in 'making maths fun' it might not broaden the child's horizons or require them to think more deeply about maths. An 'accelerated' programme might push children further through the curriculum, but without adequate guidance, this may become merely an exercise in improving or speeding up mechanical skills.

The Revised Curriculum advises giving 'more difficult and taxing problems to solve rather than prematurely pushing forward' and presenting challenging multi-step and/or open ended problems rather than overusing rote computational exercises (DES, 1999, p. 26).

It should be noted that good practice for exceptionally able students is also good practice for all students and can improve the quality of teaching and learning throughout the school. Fortunately, the primary school teacher, despite the absence of training programmes in the field, is more aware of the exceptionally able child and the 'flexible' curriculum enables the development of the abilities of all children to their fullest potential.

According to the NCCA (2007) *Exceptionally Able Students - Draft guidelines for Teachers*, there are numerous myths about exceptionally able children.

Myth 1: Exceptionally Able Children always perform well no matter what.
Exceptionally able learners may have problems like any other learner. They may have learning disabilities such as dyslexia which they can hide while the work is easier. As they progress through the school, however, it becomes harder for them to reach their potential. The ensuing frustration can lead to behavioural problems and even depression.

Myth 2: I teach mixed ability classes so I cater for all abilities.
All the activities in a mixed ability classroom can benefit the gifted child. But they certainly need an opportunity to be guided to dig deeper into topics and therefore have more meaningful learning experiences. Extra worksheets do not necessarily cater for the child's needs.

Myth 3: Exceptionally Able Children need to work with their peers.
The social development of all children is extremely important and children need to play and interact socially with other children their age. There are times, however, when they do not need to learn with them. Take, for example, the case of an exceptionally able child who has a chronological age of six and a mental age of 11 and has been reading since two. To put that child in a reading class with other six year olds who are just learning to read might be demotivating for that child. The child might also be gifted in only certain aspects of the curriculum – talented at maths but not languages.

Myth 4: An exceptionally Able Child should be considered lucky
Very often exceptionally able children can feel isolated and misunderstood. They may have more adult tastes in music, clothing, reading material and food. These differences can cause them to be shunned and even abused verbally or physically by other students. Talented children may have poor social skills.

Concluding Comment

> 'A society can claim success in eradicating the malady of mathematics illiteracy if and only if all its progeny are able to develop to their fullest potential. If its offspring can become employable workers, wisely choosing consumers and autonomously thinking citizens who can be contributors in the super symbolic quantitative world they will inherit, then society can say 'Victory is ours!''
>
> (Elliot and Garnett, 1994, p.15)

7

Some Findings from an INTO Survey on Numeracy (2011)

Introduction
The Education Committee circulated a questionnaire on Literacy and Numeracy to a random sample of INTO members in order to ascertain the views of teachers concerning literacy and numeracy and to obtain a snapshot of current practices in place in schools. Questionnaires were distributed to 1,000 teachers randomly selected from the INTO database early in 2011. The response rate was 34%, and a summary of the results that specifically relate to the Numeracy aspect of the questionnaire are presented in this chapter.

Teacher Profile
As could be expected, 89% of the respondents were female. In relation to length of teaching experience, almost half of the respondents had less than 10 years teaching experience while about 15% were teaching for between 10 - 20 years. Just less than a quarter of the replies were from teachers with 21 – 30 years of experience and the careers of the remaining 11% spanned more than 31 years.

School Profile
There was a relatively even spread of teachers in schools with fewer than 100 pupils (23%), schools with 100-200 pupils (22%) and schools with 200-300 pupils (22%). Large schools – those with 300 – 400 pupils – accounted for 13% of respondents and 17% of respondents taught in school with over 400 pupils.

Approximately 60% of the teachers taught in schools with fewer than 15 teachers, 30% taught in schools with 16 – 25 teachers on staff. Slightly more than 10% had 26 or more teaching colleagues. The majority of teachers (72%) worked in mixed schools with the remainder teaching in all-boys' schools, all-girls' schools or junior mixed/senior girls' schools in almost equal numbers.

Altogether, 30% of respondents were from DEIS schools. Very few of those who indicated that they were teaching in DEIS schools specified whether their schools were Urban Band One, Urban Band Two or Rural. A small number of teachers (6.5%) taught through the medium of Irish.

Class Details
The majority of the teachers who participated in the survey were class teachers teaching single grade classes. A total of 96 (28%) teachers worked with multi-grade classes, 18 of whom had more than 2 grades. The remaining 17 teachers (5%) were involved in learning support or special classes, and one respondent was HSCL coordinator. In all, 27% of respondents taught Infant classes.

Approximately one third of the respondents indicated that members of the middle management team in their schools held posts with curricular responsibility. Approximately one third held such posts of responsibility – with 34% having responsibility for maths.

Classroom Practice
Respondents were asked to note which methodologies they employed in the teaching of maths (see Table 1 below). The most popular method used by teachers was 'Talk and Discussion' (92%), followed by 'Active Learning' (86%) and 'Problem-solving' (83%).

Table 1: Methodology Used in maths teaching

Methodology	**Percentage of Respondents using the methodology**
Talk and Discussion	92%
Active Learning	86%
Problem solving	83%
Estimation	71%
Collaborative	70%
Using the environment	61%
ICT	60%
Maths trails	17%

When asked about discretionary time, the vast majority of teachers who replied (96%) agreed that discretionary time should be retained. Most respondents who taught 1st to 6th class considered that at least 2 hours of discretionary time would be appropriate while the majority of infant teachers (70%) considered that 1 hour discretionary time would be appropriate for their class level.

Assessment
The majority of teachers used Assessment for Learning (AfL) in maths, with 70% of respondents stating that they had used AfL that year. The majority of teachers also stated that they assessed the learning of their pupils at the end of a topic in maths.

Table 2: Assessment at the end of a topic

Subject	**Always**	**Sometimes**	**Rarely**	**Never**
MATHS	65%	33%	2%	0.3%

Standardised testing in maths took place in the majority of classes (79%) with the majority of testing taking place in May or June. Teachers reported the results of these tests to parents orally (29.7%) and in written form (22.9%), and sometimes both (22.4%). A minority of respondents (11%) stated that that they did not report the results of standardised tests to parents at all[30]. Overall, the results of tests were reported in a wide variety of forms and sometimes in more than one form. See table below:

Table 3: Format of Reporting Results to Parents

Type of Score	**Percentage of Respondents using this form to report to parents**
STEN	56%
Standard Score	12%
Percentile	24%
Descriptors	37%

The majority of respondents (76%) stated that the results of standardised tests were discussed at staff meetings and were analysed at whole-school level. Half the respondents (50%) stated that the results were plotted and compared to the normal Bell Curve. Less than half the teachers (41%) surveyed had a 3-5 year plan to monitor standardised test results with a view to improving scores. Three quarters of respondents (74%) stated that the results of standardised tests were used to inform planning for teaching. The test results assisted teachers with differentiation, grouping and identifying individual needs, particularly when tests were administered in the autumn. They also helped in prioritising curricular areas for particular attention or for selecting children who may have required learning support. They provided information to a teacher at the start of the year and many teachers reported that standardised test results helped to identify aspects of the curriculum where children were experiencing difficulty or where there were gaps in children's learning.

In total, 57% of teachers stated that they discussed each pupil's progress with him/herself and more than half of the respondents indicated that they encouraged their students to be involved in the evaluation of their own work. Respondents provided information on the range of techniques they used for self-assessment. Emphasis was placed on 'Estimate, Check and Correct'. Speed drills were also used where the children compared their results from daily/weekly tests of Tables, or Mental Maths quizzes and evaluated their progress. Work cards which have accompanying answer cards were often used for high achievers or quick finishers. ICT was also used to encourage children to compete against themselves and note improvement. In addition, children were also encouraged to correct each other's work sometimes using calculators.

[30] This survey took place prior to the issuing of Circular 56/11

A significant number of teachers kept samples of their pupils' work or portfolios, with samples of maths work being kept by 59% of respondents. In general, teachers shared the knowledge they gained from both standardised and non-standardised tests with the next teacher when handing over their class (94%). A small majority of teachers (57%) also involved students in discussion about their progress in maths.

A variety of practices existed regarding the storage of test results. In some cases the full test booklet was retained while sometimes only the cover of the booklet with the scores was kept. The majority of teachers kept tests results for one year after the test but there were many examples of the scores being held on record until children reached between 15 and 21 years. In some schools only 'unexpected scores' were recorded while in others only the resource teacher kept the booklets and then only for children with IEPs.

Of those teachers to whom the question was applicable, 97% stated that screening of pupils by the class teacher or LSRT took place in the infant classes.

Materials / Resources

A significantly large number of teachers (82%) continued to use workbooks as teaching resources in maths.

ICT formed an integral part of teaching with 80% of teachers considering it a useful or very useful tool. ICT was used on a daily or weekly basis as an integral part of their teaching by a majority of respondents, as indicated in the table below.

Table 4: Frequency of use of ICT:

Frequency of Use	Daily	Weekly	Monthly	Less often
MATHS	42%	30%	12%	12%
ENG Literacy	42%	35%	10%	10%
IRISH Literacy	25%	24%	13%	28%

Homework

Homework was seen as 'Valuable' or 'Very valuable' for pupils' learning by the majority of teachers (83%). In addition, teachers expected children to spend up to 30 minutes on English and Irish literacy and maths. Only 1% of respondents set homework for weekends. These findings are illustrated in the table below.

Table 5: Amount of time teachers expected to be spent on homework:

Amount of time to be spent on Homework	ENGLISH	IRISH	MATHS
<= 5 minutes	4%	29%	12%
6 – 10 minutes	38%	37%	39%
11 – 15 minutes	30%	23%	27%
16 – 20 minutes	23%	10%	19%
21 – 25 minutes	1%	1%	-
26 – 30%	3%	-	2%

Maths homework was generally based on reinforcing class work by doing follow-up written activities. There was also widespread emphasis on learning 'tables' or the memorisation of number facts. Problem solving, advocated as an important aspect of learning maths, formed a small part of maths assignments for home.

Learning Support / Resources

In total, 95% of teachers who responded to the question on learning support stated that they had pupils in their classes who received support from the learning support / resource / EAL team. Of those that responded, 90% stated that pupils were withdrawn in small groups and 50% stated that they had in-class support. Although the Learning Support Guidelines advocated support for children who were achieving under the 12th percentile, the teachers surveyed indicated that a large number of children in their classes with higher percentile scores than that availed of learning support – 42% and 40% of pupils receiving learning support in reading and maths respectively scored higher than the 12th percentile. See table below.

Table 6: Percentile Scores of Pupils receiving Learning Support

Percentile score at which children receive LS	ENGLISH READING	MATHS
> 10th percentile	32%	36%
11-12th percentile	26%	25%
13 - 15	11%	10%
16-20	4%	18%
21 -30	24%	9%
30+	5%	4%

However, some teachers also reported that not all children entitled to learning support availed of it. For example, 14% of respondents indicated that children entitled to learning support in maths did not avail of it. While the Learning Support Guidelines also advised that children should not remain indefinitely in learning support, 93% of teachers reported that there was no maximum number of years that children could avail of learning support.

Continuing Professional Development (CPD)

The recent Literacy and Numeracy strategy placed great emphasis on the importance of CPD for teachers in raising national standards in reading and maths. In the survey, which took place prior to the launch of the strategy, respondents were asked about their involvement in professional development during the previous 3 years and how they rated their experiences. In the first instance, it should be noted that primary teachers have a strong tradition of undertaking voluntary CPD in large numbers every year. The table below illustrates the types of CPD in which respondents engaged relating to numeracy and maths within the previous three years, and how positively they rated their experience.

Table 7: Engagement in CPD by Respondents:

Type of CPD	**MATHS** No. of teachers	**RATING** % indicating a Positive / Very Positive experience
Professional debate with colleagues in school	250	99%
Reading professional articles	114	98%
On-line courses	99	84%
Debate with other teachers at seminars/conferences	96	94%
Work with Cuiditheoir in own classroom	70	90%
Courses in local education centre	55	96%
Post Graduate studies	19	94%
OTHER	5	83%

While all forms of professional development gave rise to a high percentage of 'positive' or 'very positive' responses, those teachers who engaged in professional debate with colleagues in school rated their experience as particularly worthwhile. Reading professional articles, courses in local Education Centres, Post-graduate studies and attendance at conferences and seminars were also considered very worthwhile. This type of collegial interaction with colleagues either at staff/planning meetings provided the widest forum for staff development and strongly supports the idea of learning communities in schools. Teachers were also asked to indicate in what areas they would welcome additional CPD by ticking a prepared list of topics and subjects. The table below outlines their responses.

Table 8: Suggested areas for CPD - Numeracy

Topics for future CPD	**All teachers %**	**Teachers of Infants %**	**Teachers in DEIS Schools %**
Teaching children with difficulties in maths	70	74	29
Challenging high achievers	57	53	29
Teaching higher-order skills	45	34	28
OTHER	8	9	30

Included under the classification 'OTHER' was a wide variety of topics including assessment, classroom management, learning support in the classroom, differentiation, spelling, English as an Additional Language (EAL) support for class teachers, ICT, different writing genres, problem solving, social skills, SESE, multi-grade teaching and the development of a variety of maths skills. The need for CPD other than that which was listed was highest among the teachers in DEIS schools.

Concluding Comments

Teachers surveyed were invited to suggest changes in their school or in the wider system which could enhance both literacy and numeracy. The majority of respondents identified an overcrowded curriculum as problematic. There was a general call for a skills-based curriculum that was both challenging and relevant, more group work and peer tutoring to encourage independence, less quantity and more quality, focussed programmes for all schools, an emphasis on self-expression and different genres of reading and writing and increased use of IT.

Respondents also referred to the important role played by early screening and intervention in the development of literacy and numeracy. Respondents recommended increased in-class support, a more targeted approach in infant classes with perhaps the addition of classroom assistants, an extension rather than a decrease in support for EAL children and additional places in reading schools.

The allocation of more time for literacy and numeracy instruction also featured strongly with a small number of respondents suggesting the removal of some subjects from the curriculum altogether – subjects mentioned included drama, religion and 'extra-curricular' activities like choir and sporting activities. Some respondents suggested that it should be mandatory to spent extra time at literacy and maths especially in multi-grade classes.

Large class sizes were repeatedly mentioned as the main reason for poor literacy and numeracy skills. There was one suggestion that more movement of teachers through the school would give teachers a better picture of the 'total' curriculum. Teachers moving from mainstream to LSRT[31] and back again would also provide a wider skill basis for teachers. In addition, improved school planning, relevant CPD at staff meetings and a return of School Development Planning were mentioned by many.

Finally, it was interesting to note that many teachers commented that there were distinct signs of improvement in their schools in the areas of literacy and numeracy. There was a view that programmes such as *Jolly Phonics*, *Reading and Maths Recovery* and *First Steps* were having a positive effect on literacy and numeracy levels in their schools. Parental involvement was also seen as vital and it was strongly recommended that parents be encouraged and made aware of the importance of their role in the education of their children.

[31] Learning Support and Resource Teaching

8

Conclusion

A discussion of numeracy in the primary school should be considered in the context of the mathematics curriculum for the primary school. The desire to have a numerate or mathematically literate population is met by sowing the foundations of mathematics and numeracy in the early years and through primary education. Mathematics is part of everyday life. Being numerate is about being competent in dealing with mathematically related aspects of life such as shopping, following directions, reading timetables and solving problems.

Teachers are generally satisfied with the current mathematics curriculum, but it is clear that they are of the view that the time guideline recommended in the Primary School Curriculum in 1999 is not sufficient with the majority of teachers spending more time than the recommended guideline on mathematics. The National Strategy to improve Literacy and Numeracy (2011) acknowledged the need for more time to be spent on mathematics than the curriculum guidelines recommended. Circular 56/11, issued by the Department of Education and Skills following the publication of the strategy, recommended that 50 minutes be spent daily on mathematics. Mathematics, however, is integrated with many other subjects across the curriculum, for example collecting data as part of SESE.

Studies of mathematics in the primary school such as the National Assessments (2004, 2009) and TIMSS (1995, 2011) indicate that there are particular areas of the mathematics curriculum which pupils in Ireland have not mastered as well as pupils in other countries have mastered. Problem-solving and reasoning skills need further attention. Shape and space is also a challenge for many pupils in Ireland. Ireland also has a lower percentage of pupils than the OECD average performing at the highest proficiency level, indicating that more able pupils may not be sufficiently challenged at present. The Growing Up in Ireland Study (GUI) (ESRI, 2012) highlighted the significant role schools play in developing pupils' competency in mathematics. This finding is of particular relevance in areas of educational disadvantage, where pupils are less likely to be exposed to mathematics in their environment. At system level, the introduction of Project Maths at post-primary level seeks to promote a better understanding of mathematics and the primary curriculum in mathematics will be revised as proposed in the Literacy and Numeracy Strategy.

Teachers engage in both assessment for learning (AfL) and assessment of learning (AoL) as an integral part of their teaching. Standardised testing has taken a more central role in recent years, particularly since the publication of the National Strategy for Literacy and Numeracy. It is now mandatory for schools to administer standardised tests based on Irish norms to all pupils at the end of second, fourth and sixth class. Schools no longer have discretion in relation to when they administer standardized tests. Many schools administer standardised tests every year. It is important, however, to bear in mind that standardised tests offer a snapshot of what pupils know and can do at a particular point in time. Therefore, the information they provide is limited to the type of items covered by the test. They are also unreliable for younger pupils. Since 2012 the results of the standardised tests carried out in 2nd, 4th and 6th classes must be sent to the Department of Education and Skills in aggregate form. It is not clear what use the Department will make of this information. Assessment is also a strong feature of the mathematics curriculum in primary schools in Northern Ireland.

Mathematics teaching makes demands on teachers' mathematical knowledge (Delaney, 2010). Teachers have a strong tradition of engaging in professional development. In the INTO survey on literacy and numeracy (2011), a large majority of teachers indicated that they would like to engage in professional development relating to supporting children with difficulties in maths. More than half the teachers also indicated that would like more support with assisting high achievers in mathematics. Appropriate differentiation in large classes continues to be a challenge. Some children experience learning difficulties in mathematics while others are mathematically gifted. The challenge for the teacher is to plan an inclusive approach to mathematics in the classroom. When it comes to learning support, literacy takes priority, with fewer schools offering learning support in mathematics.

Mathematics has a language of its own, which needs to be taught explicitly to children. There are many words, or terms, that have more precise meanings in mathematics than they do in English or in Irish. Understanding the language of mathematics is key to developing skills in problem solving. Many teachers in DEIS schools use programmes such as Maths Recovery, and Ready, Set, Go Maths, to assist the teaching of maths and developing pupils' numeracy skills. Assisting parents to support their children's learning in mathematics has also been a feature of the National Early Years Access Initiative Numeracy Project in the Dublin docklands – see Appendix one. Other strategies include Maths for Fun, outlined in Appendix two, and developing maths eyes.

The INTO supports the re-presentation of the mathematics curriculum in order to address the concerns of teachers as outlined in the curriculum reviews carried out by the DES (2005a; 2005b; 2009; 2010), the NCCA (2005) and the INTO (2005; 2008). Attention needs to be given to teaching mathematics in the infant classrooms, to ensure that the principles of Aistear underpin the revised curriculum. In addition to any re-presentation of the curriculum which should clarify content and outcomes, teachers need opportunities to engage in professional development in relation to mathematics teaching. Such professional development should focus on building teachers' own competency and confidence in teaching mathematics, in addition to focusing on pedagogy, particularly in relation to teaching problem-solving skills, reasoning, shape and space and measures, the use of ICT in mathematics teaching and involving parents in supporting their child's learning in mathematics. Teachers would also appreciate support in relation to teaching children with learning difficulties in mathematics and challenging pupils who are high-achievers in mathematics. Any revision of the mathematics curriculum must be accompanied by comprehensive support for teachers, in terms of resources and professional development.

References

Bagnall, B. [Internet] *Experiencing Problem Solving - Stage: 1 and 2* http://nrich.maths.org/7231 - accessed 2nd January 2013

Black, P. and Wiliam, D. (1998). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan, 80, 139-148.*

Bourdieu, P. (1984). *Distinction: A social critique of the judgement of taste.* London: Routledge & Kegan Paul.

Butterworth, B. and Yeo, D. (2004). *Dyscalculia Screener,* London: NFER Nelson

Cosgrove, J. and Shiel, G. (2012). *PISA 2009: Results for Ireland & Changes 2000.* Press Release.

Carter, Merilyn Gladys and Quinnell, Lorna (2012). Jabberwocky : the complexities of mathematical English. *Australia Primary Mathematics Classroom*, *17*(2), pp. 3-9.

Cosgrove, J., Shiel, G., Sofroniou, N., Zastrutzki, S., and Shortt, F. (2005). *Education for life: The achievements of 15-year-olds in Ireland in the second cycle of PISA.* Dublin: Educational Research Centre.

Cramer, K. & Karnowski, L. (1995). The Importance of Informal Language in Representing Mathematical Ideas. *Teaching Children Mathematics. 1 (6).* National Council of Teachers of Mathematics

Darling Hammond, L. (2000). *Teacher quality and student achievement: A review of State policy evidence.* Seattle: University of Washington, Centre of teaching and policy.

De Lange, J. (2003). Mathematics for Literacy. In B.L. Madison & L.A. Steen (Eds.). *Quantitative Literacy. Why Numeracy Matters for Schools and Colleges* (pp. 75-89). Princeton, NJ: The National Council on Education and the Disciplines

Deboys, M and Pitt, E. (1989). *Lines of Development in Primary Mathematics* (3rd Edition) Belfast: Blackstaff

Delaney, S. (2010). *Knowing what counts: Irish primary teachers' mathematical knowledge for teaching.* Marino: Institute of Education

Department for Education & Employment, (2000). *The National Numeracy Strategy Mathematical Vocabulary,* DfEE Publications

Department for Education and Employment (Britain), (1998). *The National Numeracy Strategy*, London: DfEE

Department for Education and Science (DfES) (12/00). *The National Numeracy Strategy Mathematical Vocabulary.* DfE Publications

Department of Education & Skills (2010) *Introduction to Better Literacy and Numeracy for Children and Young People: A Draft National Plan to Improve Literacy and Numeracy in Schools.* DES: Dublin.

Department of Education and Science, (1999). *Primary School Curriculum: Mathematics. Teacher Guidelines*. Dublin: Government Publications

Department of Education and Science, (2005). *Special Education Circular SP ED 02/05.* DES, Special Education Section

Department of Education and Skills (2010). *Better Literacy and Numeracy for Children and Young People - A Draft National Plan to Improve Literacy and Numeracy in Schools*. DES

Department of Education and Skills (2011). *Literacy and Numeracy for Learning and Life: The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020*. Dublin: Stationery Office

Department of Educations & Skills (2009). *Effective literacy & literacy practices in DEIS schools*. DES: Evaluation Support & Research Unit.

Dooley, T. & Corcoran, D. (2007). 'Mathematics: A Subject of Rights and Wrongs', *Beyond Educational Disadvantage*. Dublin: Institute of Public Administration

Dowker, A. (2005). 'Early identification and intervention for students with mathematics difficulties', *Journal of Learning Disabilities*, 38, (4): 324-32

Durkin, K. and Shire, B. (1991). *Language in Mathematical Education: Research and Practice*, Milton Keyes: Open University Press

Eivers, E., Close, S., Sheils. G., Millar, D., Clerkin, A., Gileece L. and Kiniry, J. (2010). *The 2009 National Assessments of Mathematics and English Reading*. Government Publications: Dublin.

Eivers, E., Shiel, G., Perkins, R, and Cosgrove, J. (2005). *The 2004 National Assessment of English Reading*. Dublin: Educational Research Centre.

Gardner, H. (1991). *The Unschooled Mind: How children think and how schools should teach.* Perseus Book Group, USA

Government of Ireland (1999). *Primary School Curriculum: Mathematics. Teacher Guidelines.* Dublin: Stationary Office

Government of Ireland (1999). *Primary School Curriculum: Mathematics*. Dublin: Stationary Office

Greaney, V. and Kelllaghan, T. (1996). *Monitoring the learning outcomes of education systems.* Washington DC: World Bank.

Halliday M.A.K. (1975). *Learning How to mean*. London: Edward Arnold

Hargreaves, A. (1994). *Changing Teachers, Changing Times: Teachers' Work and Culture in the Postmodern Age*. School Development Series: Continuum

Henderson A. (1989). *Maths and Dyslexics.* Llandudno: St David's College

Hiebert J. and Grouws, D. (2007). The effects of classroom on students learning. In F. Lester (Ed.) *Second handbook of research on mathematics teaching and learning* (pp.317-304). Charlotte, NC: Information Age Publishing.

Inspectorate Evaluation Studies (2011). *An Evaluation of Planning Processes in DEIS Primary Schools:* Department of Education & Skills: Dublin.

Jones, M. (2010). Interesting Vocabulary. *Early Years Educator,* 2 (1) May 2010

Leading Math Success – Mathematical Literacy, Grades 7–12: The Report of the Expert Panel on Student Success in Ontario

Lerner, J. and Kline, F. (2006). *Learning Disabilities and Related Disorders* (10th edition). Boston MA: Houghton Mifflin

Lyons, M., Lynch, K., Close, S., Sheerin, E. and P. Boland. (2003). *Inside Classrooms – The teaching and learning of mathematics in social context.* Dublin: Institute of Public Administration

McCoy, S., Quail, A. and Smyth, E. (2012). *Growing Up in Ireland: National Longitudinal Study of Children: Influences on 9-year-olds' learning: Home School & Community*. Department of Children & Youth Affairs: Dublin.

Mercer, N. and Sams, C. (2006). Teaching Children How to Use Language to Solve Maths Problems, *Language and Education*, 20:6, 507-528

National Council on Education and the Disciplines, (2003). *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges*. Princeton, NJ: NCED

NCCA – Helping your young child with Maths- Tips. Dublin: NCCA

NCCA and CCEA, (2006). *Exceptionally Able Student Draft Guidelines for Teacher. Dublin:* NCCA and Belfast: CCEA

NCSE (2011). *Children with Special Educational Needs: Information Booklet for Parents*, Trim: NCSE

NCSE, (2006). *Guidelines on the Individual Education Plan Process*, Dublin: Stationary Office

Neville, A. (2012). 'Dyscalculia/Specific Learning Difficulties in mathematics: Identification and Intervention in Irish Primary Schools' *REACH Journal of Special Needs Education in Ireland*. 26 (1): 3-15

Ní Ríordáin, M. (2009). The Role of Language in Teaching and Learning Mathematics. *Resource and Research Guides* Vol. 1 No. 1 2009 NCE-MSTL

O'Donoghue, J. (2002). Numeracy and Mathematics. Irish Math. Soc. Bulletin 48 (2002), 47–55

Ojose, B. (2011). Mathematics Literacy: Are We Able To Put The Math, 4 (1): 89-100

Organization for Economic Cooperation and Development (OECD), (1999). *Measuring Student Knowledge and Skills: A new Framework for Assessment*. Paris: OECD

Organization for Economic Cooperation and Development (OECD), (2006). *Programme for International Student Assessment (PISA).* Paris: OECD

Piggott, J. (2008). A Problem Is a Problem for All That - Stage: 2, 3 and 4 - *The Scottish Mathematical Council's Journal*, 38, 54-57. – online version, accessed on 22 August 2012

Piggott, J. and Pumfrey, L. (2007). *Maths Trails – Visualising*. Cambridge University Press

Pimm, D. (1987). *Speaking Mathematically: Communication in Mathematics Classrooms,* London: Routledge & Kegan Paul Ltd.

Pitt, E. (2006). *Ready, Set, Go – Maths: A practical handbook for Teachers* Unit 1, U Store, North Park, Nth Rd., Finglas, D 11

Reay, D. (2004). Education and cultural capital: The implications of changing trend in education policies. *Cultural Trends, 13, (2), 73-86.*

Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr WH Cockcroft, (The Cockcroft Report), (1982). London: Her Majesty's Stationery Office

Research on Mathematics Teaching and Learning (pp. 334-370). New York: MacMillan. [Internet – accessed 16th September 2012]

Reynolds, D. and Muijs, A. (1999). The Effective Teaching of Mathematics: A Review of Research. *School Leadership and Management*, 19, (3), 273-288

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York, NY: Macmillan

Shavit,Y, and Blossfield, H. (eds) (1993). *Persistent inequalities: a comparative study of educational attainment in thirteen countries.* Boulder, Colorado: Westview Press.

Sheil, G., Moran, G., Cosgrove, J. and Perkins R. (2010.*) A Summary of the performance of Students in Ireland on the PISA 2009 Test of Mathematical Literacy and a Comparison with Performances in 2003*. Dublin: Educational Research Centre

Steen, L. S. (1997). *Why Numbers Count: Quantitative Literacy for Tomorrow's America*, New York, NY: College Board

Straker, A. (1993). *Talking Points in Mathematics*, Cambridge Primary Mathematics

Sutherland, R. (2007). *Teaching for Learning Mathematics,* Open University, Maidenhead, England

Thornton, C. A. and Bley, N. S. (1994). *Window of Opportunity: Mathematics for Students with Special Needs,* England: National Council of Teachers

Tout, D. (2000). Numeracy up front: behind the International Life Skills Survey. *ARIS Resources Bulletin* Mar. 2000: 1-5.

Van de Walle, J. A. (2004). *Elementary and Middle School Mathematics*, Virginia Commonwealth University: Pearson Education Inc.

Westwood, P. (2007). *Commonsense Methods for Children with Special Educational Needs* (5th Edition), London and New York: Routledge

Woodward, J. P. (2008). Dialogue is important: Language in the mathematics Classroom, *Perspectives on Language and Literacy,* 34, (2), www.questia.com

Wright, R. J., Martland, J., Stafford, A. K. and Stanger, G. (2006). *Teaching Number: Advancing children's skills and strategies* (2nd edition) London: Paul Chapman Publishing

Wright, R. J., Stanger, G. Stafford, A. K. and Martland, J. (2006). *Teaching Number in the Classroom with 4-8 year-olds*. London: Sage Publications

Internet Sources

http://www.education.ie/en/Press-Events/Press-Releases/2011-Press-Releases/PR11-07-08.html http://www.ncca.ie/en/Curriculum_and_Assessment/Early_Childhood_and_Primary_Education/Primary_School_Curriculum/Primary_Curriculum_Review_PCR_/Phase_1_English,_Visual_Arts_and_Mathematics/#3 – [Internet] accessed on 2nd January 2013

http://www.ncca.ie/uploadedfiles/P_Mild_Maths.pdf [Internet] (Mathematics - Guidelines for Teachers of Pupils with Mild General Learning Difficulties – 2nd January 2013.

http://www.nctm.org/standards/content.aspx?id=26860 [Internet] accessed 27th February 2013

Maths Trails Stage: 2 and 3 - Article by Jennifer Piggott and Liz Pumfrey http://nrich.maths.org/5456 accessed 2nd January 2013

APPENDIX I

National Early Years Access Initiative (NEYAI) Numeracy Project

In 2011, funding was made available to the Early Learning Initiative (ELI) in the National College of Ireland (NCI) under the National Early Years Access Initiative (NEYAI) to develop an Early Years Numeracy project in the Docklands area in Dublin. With support from the Department of Education and Skills, the Department of Children and Youth Affairs, Mount St Club Trustees, The Atlantic Philanthropies and Pobal, this project is one of 11 innovative demonstration projects, which will influence early years policy and programmes at local, regional and national level.

The overall aim of the project is to develop a programme to improve early years' numeracy and mathematical skills in the Docklands and inner city Dublin through the provision from birth to 6 years of an integrated programme of activities, training and support for children, their parents and families, and educators (Pobal, 2011).

Objectives:

1. The numeracy levels of children in the Docklands will be on a par with the national norms;
2. Children in the Docklands will experience a seamless development of their numeracy skills from 0-6 (DES 2010b);
3. Parents will have an understanding of their children's development in Mathematics and be able to monitor their children's progress;
4. Parenting strategies, personal skills and involvement of the parents in their children's education, particularly in numeracy, will be improved;
5. Early childhood care and education practitioners' professional practice will have improved as a result of this programme and the implementation of the *Síolta* and *Aistear* frameworks, in particular in the teaching of numeracy, in setting targets, in assessing and monitoring progress, and in using assessment information to inform the next steps for learners (DES 2010b);
6. Increased awareness throughout the community of the role that community, family and school can play in promoting successful learning, particularly in numeracy (DES 2010b); and
7. In the long term, participation in higher education by the local population within the Docklands will have increased.

A consortium of 16 local organisations, led by ELI, was involved in the application process. Each member of the consortium is jointly responsible for planning, implementing and evaluating the project.

Continuous Professional Development for ECCE Practitioners

The programme, which was linked to existing training, was designed and delivered by the NEYAI Numeracy Coordinator to nine Early Years Care and Education Centres. This consisted of three training sessions, involving 11 workshops.

Training and support in implementing *Aistear, The Early Childhood Curriculum Framework* (NCCA, 2009) was incorporated into the programme. *Síolta* was used as a prompt for reflective thinking on the quality of the service provided to children, while Aistear was presented as the framework to guide teaching and learning.

End-of-year evaluation forms completed by ECCE practitioners showed that the training and support provided by ELI helped improve practice in their setting. The support provided by ELI through on-site visits and phone calls along with the training provided to staff was listed as most useful to ECCE staff. Networking with other centres along with the information provided were also useful with all centres wanting the support and help to continue in 2012-13.

The ECCE staff evaluations were very positive with 89% of the participants stating that, following participation in the workshops, they felt confident in making changes that would have an effect on the quality of their practice with children and families.

Continuous Parental Development for Parents of ECCE children

Two workshops for parents were designed and delivered at night in the NCI and in five venues throughout the community in the morning.

The first workshop, which was linked to Síolta and Aistear was held in October 2011 and fifty-two parents attended. The topics included

- Understanding of numeracy
- Assessment of numeracy

The second workshop was held in May with 32 parents attending. The topics included

- Supporting children's numeracy development
- Síolta Standard 5 Interactions
- Concepts of Shape and Space

These workshops were very well received by those parents who attended. However, recruiting and maintaining numbers was a challenge and ways of improving attendance would have to be looked at for any future sessions.

Curriculum Priority and Related Activities

The theme of the first curriculum priority week was positional language. Activities and resource packs were devised for both staff and parents with assessment for learning built in to the programme. Playing 'Hide and Seek', along with rhymes, was the main learning activity. The second curriculum priority week focussed on shape and space. Only the early years' settings were involved with the second curriculum priority week as the primary schools were involved with Zoom Ahead with Books Project. The third week had as its curriculum priority shape, space and size with finding objects of different sizes in the environment as the main activity.

ECCE staff who participated in the project agreed that it prompted staff to work together as a team to plan quality early numeracy experiences for the children. Most of the primary school principals agreed that it prompted the teachers to reflect on quality early numeracy teaching practice in the school, while the majority of the ECCE managers felt that it provided valuable learning opportunities for staff in their centres.

Overall, the Curriculum Priority Week worked well. It raised the awareness of parents and ECCE staff of the importance of early numeracy and ensured that more time was given to numeracy activities, both at home and in the ECCE centres. There was good feedback about the resources and activity guidelines. Organising the Curriculum Priority Week was, however, very time consuming for the co-ordinator and it was challenging to ensure the resources matched the needs of all settings and were age-appropriate.

Story Telling Sessions with a Mathematical Theme and Language
Two on-site workshops took place with a numeracy theme integrated throughout each workshop. The first one was based on the book *'Jack in the Beanstalk'*. The second workshop, which was delivered in the ECCE settings only, was based on the book *'The Big Red Bath'*. Each setting received a range of materials including a list of thematic songs and rhymes, along with several copies of the books, so that they could reinforce and extend the learning from the workshops.

Feedback on these sessions was positive and all agreed that the children enjoyed the experience, however, there were mixed views on whether it was the most useful way to get parents involved. There were also mixed views on whether the sessions enhanced the quality of practice in the settings.

Assessment and Evaluation
Having consulted with the National Council for Curriculum and Assessment (NCCA) and received advice on assessment methods, assessment for learning was included as a central element of all the curriculum priority materials and evaluation forms. However, more work is needed both on developing a system for the systematic collaborative assessment and evaluation of children's mathematical learning and staff training in assessment techniques. Through discussions on children's engagement in the curriculum priority there is greater awareness by ECCE practitioners and parents around assessment for learning.

Main Findings:
The programme worked well with visible changes in some centres regarding Quality ECCE Practice. Interest in training increased as ECCE managers could see the benefit of a centre-wide approach to quality improvement. Most centres now try to send all staff to the workshops and this approach has made the process of change more effective, as well as improving staff morale. The consistent requests for further 'training'/ workshops are positive as it shows enthusiasm for more knowledge and interest in improving practice. However, meeting these requests is dependent on resources and funding.

There were varying levels of engagement with Síolta and Aistear/ CPD process across the centres involved. There was also the on-going challenge of irregular staff attendance at workshops as well a very low awareness of Síolta and Aistear amongst primary school infant class teachers. Other challenges included scheduling and balancing the workload of the training sessions with the on-site support and NEYAI Curriculum Priority work.

All of the nominees agreed that the Working Group was a good forum for sharing and learning from others and found it easy to communicate the proceedings of the working group to their colleagues. The majority of participants identified the sharing of ideas with other settings as the most valuable aspect of the programme.

APPENDIX II

Maths for Fun

In the Executive Summary of the 2005 DEIS 'Action Plan for Educational Inclusion', Parental Involvement and the forging of closer ties between the school and the wider community is listed among the areas that had already been receiving significant emphasis since the late 1980s. Successful home/school/community linkages and greater parental involvement in school life had been emerging and this was noted among the existing strengths in the approach to addressing educational inclusion issues up to 2005. The Action Plan, among its many and varied aims, sought to strengthen supports for parental and family involvement in education, through targeted measures to tackle problems of literacy and numeracy, with particular reference to family literacy. In its view of reform, it was agreed that there was a need to improve integration of educational inclusion measures and to enhance delivery structures to further strengthen the involvement of parents, family members and the wider community.

Literacy and Numeracy were acknowledged as fundamental skills in deriving benefit from education and the proposed support measures included further development of Paired Reading and Paired Maths. In recognition of the vital role of parents as the prime educators of their children, it also proposed further support for 'Reading for Fun' and 'Maths for Fun' initiatives, implemented through the HSCL Scheme, as these initiatives directly involved parents and other family members in classroom or home-based activities, assisting children's literacy and numeracy development.

In highlighting the role of home and the local community, the Action Plan refers to the 1997 OECD study, 'Parents as Partners in Schooling', which cited the HSCL Scheme as 'a good example of innovative central government initiatives', and stated that, 'It is clear from the Irish experience that educational initiatives, based in schools, can raise the educational level of the adults involved and result in a general sense of empowerment in the local community. Parental involvement, especially in areas of socio-economic deprivation, does not just benefit the children in the school, it is a crucial aspect of lifelong learning.'

In late 2012, a general request was made of HSCL Coordinators across the country to share whatever views, or experiences, they had on initiatives that involved parents in numeracy activities.

In the main, responses referred to the continuing use, after so many years, of 'Maths for Fun'. In testament to its durability and on-going effectiveness as an inclusive project, those who continue to use it are as enthused by it and as committed to it, as ever. Local conditions and contexts may not have allowed for everyone to experience the initiative, but for those who have, it has always proven highly enlightening and successful, particularly as an educationally inclusive 'gateway' towards allowing early, non-threatening and guided participation by parents, who for whatever reason, may lack the confidence, or self-belief, to become active participants and partners in their children's educational journey.

In the context of early stage parental involvement in numeracy in school, the transformational potential derived from a parent engaging with a group of young children, directing the simplest of games can never be underestimated, both for the parents involved and the children's perception of their parents.

'Maths for Fun': An Overview

'Maths for Fun' is a collaborative learning initiative, designed and structured to address the following aims:

- To meet the individual needs of the pupils through hands-on work and parental involvement,
- To enhance the parent-teacher partnership,
- To help parents understand more fully the challenging nature of maths for the pupil, as learner, and the teacher, as tutor,
- To break down the fear barrier some pupils may have towards maths, by bringing fun and variety into the learning process,
- To help parents experience, at first hand, the working school environment,
- To empower parents to engage meaningfully in the learning process of young people.

It provides the opportunity to target particular parents who, as students themselves, may not have fully engaged with the educational process and may not have a sufficient level of awareness and competence, when it comes to working with their children and maximising their opportunities at school. 'Maths for Fun' sessions afford parents and teachers the opportunity to familiarise themselves, on a human and professional level, with each other and allow the parents, through the various activities, to build confidence, competence and a sense of partnership with the teacher and the school.

The activities used in 'Maths for Fun' can vary greatly and can involve expensive, professionally-produced materials or simple hand-crafted games, created within the school, at minimal expense. They can include anything from tangrams and pattern blocks, aimed at developing spatial awareness, to dominoes and games for developing number and computational skills. The activities can also involve relational attribute blocks, aimed at developing language, logical thinking and problem-solving skills. Clock bingo can help consolidate work done in class on time, and pentominoes can help develop problem-solving skills and the concept of patterns and tessellation.

'Maths for Fun' sessions take place over a fixed number of weeks, usually four to six, and each session lasts about an hour. Depending on the size of the class and the number of available parents, the class is divided into groups, with no more than five pupils in any group, if possible. A parent takes charge of a particular maths activity. Children move from one activity to the next, at a given signal. The class teacher, if not needed to take an activity, maintains a supportive role within the room, while the HSCL generally oversees the process and reviews the session, later, with the parents.

Activities

Because the games used in 'Maths for Fun' can be of a general nature, or specific to a particular strand of the maths curriculum, the choice of activities is very broad. Sometimes the teacher will come up with an idea that best tackles the problem area. Many have found that simple dice and card games have worked particularly well and the simple activity of sharing out cards and playing simple games like 'Snap', 'Go Fish' or 'Pig' can work wonders for a child's understanding of socialising, taking turns and the need to concentrate.

Games and Activities with Dice (Claire Publications: ISBN 1871098432) provides many very worthwhile options for all ages and really gets the children thinking about number and counting. For the reluctant parent, simple, familiar games such as 'Connect 4' or basic card games can provide early encouragement and engagement with the process.

Conclusion
'Maths for Fun' is an uplifting experience for parent and child and helps develop numeracy awareness in the home. Connecting parent and child in experiencing numeracy together usually transfers into positivity in the homes of all pupils, particularly those of a marginalised and educationally disadvantaged background.

The very process of initial contact with the parent, introducing the proposal, the guided preparation, training and reassurance before engagement with the children ensures a safe environment for parents to take initial steps into involvement in their children's education. It affords them the opportunity to re-engage with the education process through their children and see school as a welcoming place, in which they have a hugely important role to play.

The original model for HSCL allowed co-ordinators to take the time and sensitivity necessary to succinctly draw their more marginalised parents into the education process through initiatives such as 'Maths for Fun', however, given the increasing constraints and re-shaping of the co-ordinator's role, it would be most unfortunate if, in future, they were restricted in providing support for this time-tested initiative that has served so well to support Parental Involvement, that 'crucial aspect of lifelong learning', in areas of socio-economic deprivation, for over thirty years.

Part 2

Proceedings of the Consultative Conference on Education

15 & 16 November 2013 Portlaoise

6

Presentations

Welcome and Introduction

Dympna Mulkerrins, Cathaoirleach, Education Committee

A Chairde agus a Chomhmhúinteoirí,

I would like to add a few words of welcome to our guests and delegates here today. The Education Conference plays a very important role in the work of the INTO. It is at this conference that we get to discuss and hear the views of teachers on many education issues. In preparation for the conference the Education Committee carries out research on the topic being discussed and presents these finding in the background document, which was sent to all delegates.

After the draft plan to improve literacy and numeracy was published, the Education Committee carried out a survey in 2011. We presented our findings on literacy at our Education Conference two years ago. We now include the findings on numeracy in this year's document. What we were most interested in was whether much had changed in the teaching of numeracy and maths in the meantime – particularly since the publication of the National Strategy for Literacy and Numeracy. So we prepared an online questionnaire - which gave us some interesting additional information.

The Department will be pleased to know that 99% of respondents are familiar with the Literacy and Numeracy Strategy! Has the Strategy had an impact? Well three in four respondents said that it impacted on the school plan for maths. Three in four also said that they had increased the time they now spend teaching maths. This time is taken from discretionary time – 60% of respondents, by integrating maths with other subjects - 38% of respondents, and by reducing time spent on other subject – 39% of respondents. Interesting! Most of the additional time is spent on number which was prioritised by 90% of respondents. Two thirds indicated that they spend more time explicitly teaching the language of maths.

I think what is particularly noteworthy is that pupils' enjoyment of maths has increased. The Department will also be pleased to know that the vast majority of teachers (85%) are aware of the School Self-evaluation template for reviewing mathematics, though less than half the respondents have actually used it in their own school. On the other hand, more than half the respondents have reviewed maths as part of their self-evaluation process.

It emerged that two thirds of teachers give more challenging work to high-achieving students, and one in six teachers give those pupils extra work. This is a positive finding as national and international surveys suggest that our high-achieving pupils are not sufficiently challenged in our system and teachers can find this area difficult to manage.

Here are some other interesting findings:

- parents are rarely invited into classrooms to support children's learning
- four out of five teachers present a maths-rich environment
- five in six teachers would like professional development in teaching problem-solving
- two-thirds of teachers have discussed maths teaching during Croke Park hours

Our thanks are due to all those teachers who took the time to complete the questionnaire and give us their views.

With all these issues to debate and think about we should have some good discussion groups.

You're also being invited to complete the Maths Quiz prepared by the Education Committee. This was on your seat when you arrived in the hall. When you have it completed put it in the box at the registration desk. All correct entries will be put into a draw and a winner picked at tomorrow's plenary session.

Enjoy the conference.

Deirbhile Nic Craith, Senior Official

Ba mhaith liom cur leis an bhfáilte a chuir an tUachtarán agus cathaoirleach an Choiste Oideachais romhaibh anseo inniu. Is mór an taitneamh a bhaineann baill as an gComhdháil Oideachais Bhliantúil.

Chomh maith leis an bplé ar uimhearthacht is ocáid stairiúil í an Chomhdháil seo. Beidh iris á seoladh againn um thráthnóna – iris téagartha ina bhfuil ábhar machnaimh scríofa ag múinteoirí do mhúinteoirí. Deis iontach í an iris seo do mhúinteoirí torthaí a gcuid taighde a roinnt linn. Is ró-mhinic a fhanann taighde a dhéanann múinteoirí ar an seilf. Is mór an onóir dúinn an tOllamh Emeritus John Coolahan a bheith linn níos déanaí chun an iris a sheoladh.

We'll have more about the Journal later. I note, however, that we do not have an article on maths in this edition. There may be some delegates here today who are carrying out research in this area and we look forward to receiving articles from you in the future.

The purpose of my presentation is to offer some context – both national and international – for our current focus on numeracy.

Numeracy is receiving a lot of attention in recent times. This is not surprising given that having a population who is numerate is considered important for the advancement of a country's economic and social wellbeing.

According to the OECD, "today's labour force has to be equipped with the set of skills and competencies which are suited to the knowledge economies" (OECD, 2009). Key capacities for economic success in the 21st century include information-processing competencies defined in terms of proficiency in literacy, numeracy and problem-solving in technology-rich environments (*OECD Skills Outlook 2013)*.

These are the skills assessed in the recent Survey of Adult Skills.

The findings of the recent Adult skills survey show that "what people know and what they can do with what they know has a major impact on their life chances" (OECD, 2013, p.26). Lower skilled individuals are increasingly likely to be left behind and countries with lower levels of skills risk losing competitiveness as the world economy becomes more dependent on skills.

While success is increasingly about building skills beyond formal education (p.34), the OECD identifies the following points for policy which are relevant to us in primary education – essentially, to provide high-quality initial education and to make sure all children have a strong start in education.

Primary education, therefore, plays a crucial role in laying the foundations for the knowledge, skills and competencies that people will need – not only in work but in their life. Primary education is, of course more than that. The child's holistic development and well-being is a core element of our primary school curriculum – as we discussed at last year's conference.

Ireland is not immune to global developments in education. But, as Ciarán Sugrue said, policies and discourses are 'refracted' differently across countries. So Ireland has its own way of incorporating international developments into local policy.

The globalisation of education has led to The Global Education Reform Movement (GERM), leading to many similar education reforms across countries. School self-evaluation is the current policy travelling across countries!

At the centre of the global education reform movement is a focus on 'core subjects' and the targeting of proficiency in literacy and numeracy. It is no surprise, therefore, that there is a strong focus on developing literacy and numeracy in Ireland at present. Along with a focus on core subjects, which includes demands for increasing instruction time for 'core subjects', the global education reform movement is associated with increasing standardisation in education, teaching with prescribed curriculum to reach predetermined learning goals, the transfer of models from the corporate world as the main logic of change management and high-stakes accountability policies. To what extent Ireland is influenced by these global trends is a matter for discussion – perhaps at a future education conference - but I suspect the Literacy and Numeracy Strategy was influenced by these trends. Such trends have major implications for our education system.

As part of this global education reform movement there is a fascination with measuring or assessing the skills of adults and young people.

After an absence of 16 years, Ireland participated in TIMSS in 2011 and intends to do so again in 2015. TIMSS is an international assessment of mathematics and science skills, for pupils in 4th class. Both Ireland and Northern Ireland did well in TIMSS 2011.

Ireland has participated in PISA since it was established in 2000. PISA is designed to reflect the competencies in literacy, mathematics and science, required for success in the 21st century. PISA is for 15 year olds, which is at the end of compulsory schooling approximately. Ireland was significantly below average in mathematics in PISA 2009. The 2012 results - available in December – are eagerly awaited.

Where adults are concerned, in 2012 Ireland participated in PIACC – Programme for International Adult Competencies – also known as the Adult Skills Survey. While average in literacy, Ireland was below average in numeracy. Problem-solving is consistently identified as a challenging area for us.

PISA seeks to identify ways in which students can learn better, teachers can teach better, and schools can operate more effectively; the Survey of Adult Skills focusses on how adults develop their skills, how they use those skills, and what benefits they gain from using them. The value of international comparisons depends on how they are used.

The Literacy and Numeracy Strategy signalled Ireland's major programme of reform in education – as it referred to teacher education, curriculum content, the assessment and reporting of student progress, and evaluation and assessment policies. And as Dympna Mulkerrins has outlined the Strategy is having an impact in schools. There is no doubt that Ireland wishes to enhance the competency of its citizens in both literacy and numeracy. This is reflected in all recent policy development.

The media certainly like to remind us when we are not so good. An article in last week's Irish Times suggested that we have a cultural bias against maths, and article to which Seán Delaney will also refer tomorrow.

However, we do have reasons to celebrate. The recent report published by the Inspectorate indicates that 85% of mathematics lessons observed were satisfactory or better. If this were the Leaving Certificate this would be an A grade.

> Standards in teaching and learning of mathematics were generally good at primary level with learning satisfactory or better in 85 per cent of lessons inspected and good results for Irish students in international surveys. However, inspectors noted the lack of opportunities for primary pupils to engage in the sort of collaborative working needed for effective problem-solving.
>
> (Department of Education Inspectorate, 2013)

In general, the Inspectorate's report was very affirming for teachers. We can do better, however, in assessment, particularly formative assessment, the use of ICT and opportunities for collaborative learning.

So where are we going? It was flagged in the Literacy and Numeracy Strategy that there would be revisions to the curriculum. We expect to have a revised Language Curriculum – English and Irish – for infants to second class available for consultation next Spring. Work on revising the maths curriculum is also starting this year. As part of its preparation the NCCA commissioned a research report on teaching mathematics to young children. And I'm delighted that Therese Dooley, one of the key authors of the report, will be addressing you this afternoon. It's an excellent report which will influence developments in the maths curriculum. It will soon be available on the NCCA website. One of the issues to be discussed in your groups today will be changes, if any, that should be made to the current maths curriculum. Second level have seen the development of Project Maths – not without controversy – which seeks to move away from rote learning to building a knowledge and understanding of mathematical concepts and processes. A copy of the bridging framework which shows the links between the primary curriculum and the junior cycle curriculum is included in your packs.

Bainigí sult agus tairbhe as an gcomhdháil.

Introduction and Overview

Anne English, Education Committee

While the ability to use mathematics to solve problems and meet the demands of day-to-day living has always been a central focus in our primary schools, this key attribute of numeracy has been afforded heightened priority in our educational system of late. This is not surprising, since our National Strategy to improve Literacy and Numeracy stresses that without the skills of numeracy, a young person is cut off from contributing to many aspects of the society and culture in which they live ... A sobering thought indeed.

In response to the Literacy and Numeracy Strategy, and requirements related to School Self Evaluation (SSE) and the School Improvement Plan (SIP), school communities are reflecting on practices around teaching and learning in schools and working towards maximising children's learning in numeracy.

Now, there are teachers here today from all corners of Ireland. While we have much in common with each other in terms of the school-related challenges that we face on a daily basis, we work in a variety of vastly different contexts, ranging from large inner-city schools to small schools in rural areas, for example. And, of course, each class of pupils is made up of unique individuals with their own unique needs. Nevertheless, in spite of the diversity of school contexts in which we work, it is noteworthy that the responses to the INTO's recent survey on Numeracy in Primary School, a significant number of teachers highlighted the same areas as priorities for development and growth in their schools:

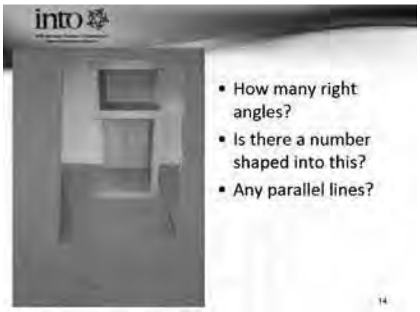
- Making maths meaningful – linking it with practical living and the environment and extending it beyond the maths lesson ... bringing it alive
- Reducing reliance on textbooks
- Greater focus on problem solving
- Developing the language of maths

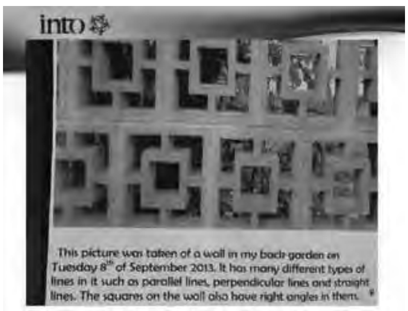
... all of these featured prominently in teacher feedback.

Effectively, there is a strong message coming through that teachers are keen to ensure that the children see, use and talk about maths as a natural part of their lives and that numeracy learning is embedded in the child's day-to-day living. Concepts, skills and procedures that relate to everyday problems must not be an optional add-on or extra to the maths lesson - something to fit in when we get the chapter in the book 'finished' and if time allows. Rather, the child's real environment must inform how we approach the teaching of numeracy from the outset. Similarly, problem solving must be something that is real and meaningful to the child and must reflect the reality of problems with which the learner is likely to be faced in the real world. We are talking about forging a strong link between what and how we teach in the classroom and the problems and challenges that the learner comes up against in life.

How do we advance this work? Well, the classroom of today is an extremely busy place. When seeking to enhance children's learning in Numeracy, rather than add on further programmes , we need to look at ways of 'tweaking' or streamlining what we do already, or embracing projects that demand little time or resources relative to their overall benefit. With this in mind, I am now going to direct our attention briefly to two key areas - the Maths Eyes project, first of all, and then problem solving. This will not be a comprehensive treatment of either of those areas – rather some food for thought as we get started at our Numeracy Conference.

Maths Eyes has its roots in a successful initiative that got underway in Tallaght in 2011. *(See www.haveyougotmathseyes.com for fuller information on the background to this project).* In a nutshell, it centres primarily on encouraging people to make the link between the Maths they do and the real world. As you've probably noticed already, this aligns very nicely with the priorities expressed by teachers who took part in the INTO survey on Numeracy. Now, time doesn't allow for a rich description of the entire project – so I'm just going to give you a glimpse of how we embraced the Maths Eyes project in our own school. We asked that all children in the school, would take a photo of something in their environment that relates to maths – with the specific theme of shape, line and angles in mind. They were also asked to write a caption, or labels or questions alongside the picture. This prompted them to think a little more deeply about it and to engage with the language of maths – it also facilitated the school in creating a display that would promote the mathematical thinking of others in the school. As you can see from the sample photos here, a wide range of material was returned by pupils of all ages.





It really created a terrific buzz around maths in the school and scrapbooks made up from the material will be a useful resource in the school in the months to come. The beauty of the project is its simplicity – the children, together with their parents, create the resources and in so doing they engage themselves more fully with the maths programme and with maths in their environment. The children genuinely developing Maths Eyes.

And the whole project generates a very positive vibe around maths, without a textbook in sight!

I'll briefly touch on the area of problem solving now. There are many definitions and descriptions of problem solving available, but the following key characteristics crop up again and again:

- problem solving involves overcoming obstacles,
- it involves engaging in a task for which the solution is not known in advance,
- it can be perplexing and difficult, and
- it may take quite some time to come up with a solution,
- sometimes there is more than one 'right' answer,
- there can be many ways to solve the same problem.

While we tend to associate problem solving with the maths curriculum, in actual fact problems permeate every curriculum subject and every facet of life. It is unlikely that an approach to problem solving that concentrates solely or predominantly on 'completing' a page or a page and a half of fairly repetitive word problems at the end of a chapter in a maths textbook will optimally equip a child to become competent in the area of solving maths-based problems in the real world. Certainly, practice and repetition can have their merits, but we must ask ourselves if repeatedly applying the same formula to a page of word problems that mirror each other really is the best use of time. And in the true spirit of a problem solving approach, as in settling into a challenge that is perplexing and difficult, for which there is not necessarily an instant answer, but which various people may solve in various ways, I'm going to present some problems or challenging questions to all of you here this evening:

- How do we, with the limited time at our disposal, optimally develop the problem-solving skills and aptitudes of our pupils?
- How do we ensure that the higher-achievers remain challenged so that they will continue to develop their problem-solving skills rather than just engage in completing pages of questions based on the same formula and drill?
- How do we ensure that the child who finds maths particularly difficult, gets to engage in a level of problem solving that is manageable for them?
- How do we ensure all children will have their interest hooked by the problems presented and will be sufficiently, yet not excessively, challenged?

Unfortunately, the answers to those questions that I've just posed are not at the back of the textbook, or in this case, the Conference Document. Not having immediate answers to these questions is not a sign of failure on our part. They are questions that are not immediately answerable but that we can all collectively work on as we advance the teaching and learning of numeracy in our schools, and as we co-create professional knowledge in the proud tradition of our engagement with the issues raised in the Education Conference and beyond. Persisting in grappling with the challenges and problems that teaching and learning pose is a defining feature of our professionalism as teachers and is to the ultimate benefit of the children in our care and to society at large.

Teaching mathematics: The power of uncertainty
Dr Thèrèse Dooley, St Patrick's College, Drumcondra

Thank you very much indeed for the warm welcome and I am delighted to be here speaking about the teaching of mathematics. It is a subject about which I am very passionate. I was brought up with a love of mathematics. My mother was interested in both mathematics and baking - she encouraged my siblings and me to use multiplication, fractions, ratio, area and so on as we prepared ingredients and cooking materials. We never even realised that we were doing mathematics! In school then, I was very positively disposed to the subject – however, the system at the time did everything that it could to convince me that girls didn't need to do mathematics and weren't necessarily good at mathematics. Happily, I persisted with my studies. When I started teaching, what really surprised me was that some pupils in Sixth class did not participate to any great extent in the subject. I then went on and did some research in the area of mathematics education - particularly in pupils' engagement in mathematics. I now work in St. Patrick's College of Education and find that many student teachers have poor relationships with mathematics.

This crisis or failure in mathematics is well played out in the media and, of course, a lot of this is exaggerated as our students do not do very poorly in national and international assessments. They do well in some areas, for example, number and number operations - but a consistent finding is that students at first and second levels of education do not attain high scores in items that assess performance in problem solving and critical thinking skills (e.g., Mullis, et al., 2012; Perkins, et al., 2013). It is encouraging that there is now greater awareness of the need to engage children in mathematical problem solving and of the nature of problem solving. There is an acknowledgement that students need to move beyond textbook word problems, that they need to feel stuck and uncertain at stages of the problem-solving process. Of course, the role of teachers is crucial in this - teachers have to develop or identify rich tasks and have to set up the kind of environment where children have time to conjecture and reason and think about possibilities. However, it is also true that teachers in these situations often feel uncertain and uncomfortable themselves. I argue that this teacher uncertainty and discomfort play a vital part in developing a problem-solving environment.

Central to my talk today is a lesson that I taught to fifth class pupils. The lesson focused on the Grasshopper Problem. Some of you may be familiar with this problem. Basically it concerns a grasshopper jumping along a line of length 1 metre. He first jumps half-way along and then half-way along the remaining distance and so on. The question that I posed to the pupils related to identification of the landing points in the first instance and then to the possibility – or not – of him arriving at the end of the line. This problem is one with which mathematicians have grappled for centuries and, in fact, had to invent a new mathematical entity in their efforts to solve it. It is based on Zeno's paradox of motion[32] (O'Connor and Robertson, 1999), - from a practical perspective, the grasshopper will eventually get to the end of line but, from a mathematical point of view, the journey goes on forever (Stern and Mevarech, 1996).

The Fifth class children did quite well in identifying the first few landing points, that is ½, 3/4, 7/8 but they were more uncertain about the naming of the next landing point. This is not surprising as children in primary school do not, for the most part, encounter fractions with denominators greater than 12. What happened at this juncture was a critical moment for me as a teacher. Before talking about this, let me dwell for a very short time on the problem itself. We could ask if it is suitable for children in primary

[32] Zeno, however, referred to a continuous journey.

school especially if we are of the view that the learning of mathematics is about preparation for the future. Do children need to know about a grasshopper jumping across a line!? One thing we have learnt from the economic downturn is that we really do not know what kind of future children are facing. Given the rate of technological progress, we cannot predict with any certainty what our lives will be like in 20 or 30 years' time. Professor Bill Barton of Auckland University described, in a talk he gave at the University of Limerick earlier this year, that mathematics content actually doubles every 25 years. He argued that we are at nothing if we are trying to prescribe the content with which children need to engage and that what students at first and second levels of education need to experience are the power and pleasure of mathematics (Barton, 2013).

Why is it that, despite daily engagement in school mathematics, students are not competent or confident in problem solving or higher-order thinking? Much goes back to our views of mathematics. Traditionally the view of mathematics is that it is made up of an objective, immutable body of facts to be transmitted from generation to generation - such a view has attracted an elite few into mathematics. A more current perspective is that mathematics is a cultural activity made up of different social practices - such a view enhances engagement and participation in mathematics (e.g., Ernest, 2004). Despite the popularity of this latter view, what is pervasive in the media and in images of mathematics is its certainty. Take, for example, a fact like "three plus two equals five" – this is presented in media and in general conversations as a simple and uncomplicated truth. Behind it, however, is something very complex. The symbols, 3, 2, +, and = are cultural artefacts. They are used in some societies and in some cultures but not all. Furthermore, a number of assumptions are made. One is that we are adding like entities - for example, three apples and two apples or even three apples and two oranges. But even when we are adding seemingly like entities, the answer is not always five, for example, 3 metres and 2 centimetres. Now, I don't want to spend the next 25 minutes or so convincing you not to teach number facts in school. What is more important is the need to examine and explore situations where three and two equal five. Mendick (2006) argues that we actually need to look at who is doing the counting and why and what the consequences of the addition are. If three bananas and two apples add up to five pieces of fruit what are the consequences? Are there enough for "five a day", enough fruit to make a smoothie, does it mean the same thing in different contexts and for different children? And that is what is at the heart of real-life problem solving.

Hersh (1997) talks about the private and public side of mathematics – the public side of mathematics is like the finished cake (I go back to my baking again!), a very well-ordered and neat product as shown in journals, textbooks, lectures and so on. The private side of mathematics is where mathematicians work in a very busy but perhaps messy way – where they collaborate and think and use words like "likely", "maybe" and "perhaps". Young learners, however, are often presented with the clean version of mathematics where there is no need to struggle. Their role in the classroom is often as passive receivers of mathematics. What research has found is that this has the effect of disengaging students from mathematics as they have no agency in the learning process. The alternative view of mathematics leads to situations where children collaborate, where they try to make sense of mathematics and where errors are an intrinsic part of the process. In this kind of situation, children are active participants in the mathematics making. This kind of experience leads to greater persistence and success in the subject (Boaler and Greeno, 2000).

In research on pupils' attitudes to mathematics, a number of fifth and sixth class girls from different classes were asked what they thought a good mathematics pupil was (Cawley, 2011). Here are some of their responses:

[A good mathematics pupil ...]

- can do all their sums so fast and gets them all right
- knows how to solve problems and somebody who just enjoys doing it
- is very good at listening to what the teacher teaches them during maths class
- pays attention, never gives up on a sum, someone who feels confident
- listens and always gets good marks
- gets ten out of ten on their tests
- enjoys working with numbers and likes solving problems, and
- knows all their tables and someone who works hard and keeps trying and trying

I think what is noteworthy about these is that some children are very perceptive: they understand that maths is about enjoyment, problem solving, persistence and so on. What predominates, however, is the view that 'being good at maths' is doing well on tables, on sums and on listening to the teacher. None of the students mentioned anything about collaboration and working with others and, therefore, a sense of mathematics as a solitary, quiet activity is conveyed.

I was interested in seeing how this "busy but messy" environment could be created and therefore conducted research in three different classes (two Fifth classes and one Fourth class) in an endeavour to uncover the conditions under which students might make sense of mathematics (Dooley, 2010). Suffice to say that I collected lots of data and had a very "busy and messy" study at the end of it all!

As I mentioned earlier, I want to talk today about a lesson based on the Grasshopper Problem. I taught this lesson in the early part of my research to a Fifth class in a provincial town in Ireland. There were 30 pupils in the class - 17 girls and 13 boys - and they were what you might describe as a fairly typical class. The spread of results they achieved in a standardised mathematics test at the end of Fourth class centred around the average standard score. There were four pupils at or above the 90th percentile and five at or below under the 20th percentile. At the beginning of the lesson, I drew a line on the blackboard and I asked the children to talk about the different landing points of the grasshopper and whether he would ever reach the end of the line. I want to zone in on the contributions of two particular pupils to this lesson. One was Kate who would have been deemed to be of "below average" achievement on the basis of her test results and Jack who was of "average" attainment in mathematics[33]. As I explained at the beginning of this presentation, everything was going well until the grasshopper jumped past the "7/8" point - this is how the conversation unfolded:[34]

104	TD:	What do you think is going to happen next?
105	Chn:	It's going to half it//half it//half ...
106	Jack:	It's half of seven eighths (whisper) [ ]
125	Jack:	He's on fifteen sixteenths.
126	//Ch :	Seven and a half eighths.
127	Chn:	No.
128	TD:	He's on fifteen sixteenths, seven and a half eighths or fifteen ...

[33] Gender-preserving pseudonyms are used throughout the paper.

[34] Transcript conventions are: TD: the researcher/teacher (myself); Ch: a child whose name I was unable to identify in recordings; Chn: two or more pupils making utterance simultaneously; … : a short pause; [ ]: lines omitted from transcript because they are extraneous to the substantive content of the lesson; //encloses utterances overlapping that of next or previous speaker; (word): transcriber's comments.

//*Ch talking* … So you think he's on fifteen sixteenths. Where are you getting fifteen sixteenths from?

129 Jack: Cos I think, I think em … I think a half an eighth is sixteen …

130 TD: Right.

131 Jack: and eh …

132 //Ch: I know …

133 Jack: when …

134 //Chn: Seven and a half!

135 Jack: and then another sixteenth … if she went another sixteenth (*other children talking in background*), she'd be there but she didn't go another sixteenth, so she went fifteen sixteenths.

What I think is really interesting about this is that a child who is deemed to be of average attainment is engaging in very sophisticated mathematical reasoning and is really making sense of this problem. I then went back to Kate and asked her why she had proposed $7\frac{1}{2}$/ 8 as a solution:

143 Kate: It was seven eighths and if he went eight eighths, he would be at the end, so if you go half of it, then it's seven and a half.

Interestingly, her thinking mirrored Jacks. At this point I asked the other pupils in the class to decide on how the point should be named and they began to argue:

159 TD: … [W] ill we call it seven and a half eighths or fifteen sixteenths?

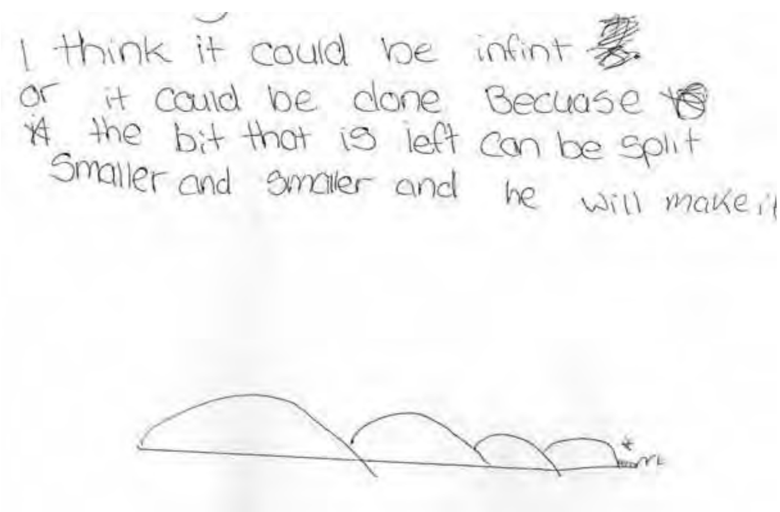
160 Chn: Seven and a half //Fifteenth sixteenths//Seven and a half is easier to manage// No, it's not//Cos you are going two, four, eight//Seven and a half is easier.

I am going to pause for a moment and ask you to think about what you as a teacher might do in this scenario. Are you feeling uncertain, uncomfortable? Yes, I did also and my discomfort is palpable in the next part of the excerpt:

161 TD: I think … you could call it seven and a half eighths but we normally these things … normally they are brought up to full numbers. But seven and a half eighths would be ok but … normally it's brought up to something like fifteen sixteenths.

I wrote both numbers on the blackboard and then after that we went on to discuss whether the grasshopper would actually reach the end of the line.

Boaler (2003, p.12) says that "teaching is a practice that takes place at the intersection of hundreds of variables that play out differently in every moment". What we, as teachers, do is very complex. Kate is the "low achiever" in mathematics who makes a very sensible suggestion. While it is not conventionally correct, it is sensible and well-considered. I know that at this point she might well decide to enter into or turn away from the world of mathematics. I had to take account of the social and personal risk she took when she made her contribution in the classroom situation. I also kept my eye on the conventionally correct "15/16" and thus I was *dancing* between the mathematical perspective and the human perspective. I was very pleased with what I did that day. Later on, in Kate's written work she wrote, "I think it could be infinity or it could be done because the bit that is left can be split smaller and smaller and he will make it."[35]



Here, it seems to me that she is thinking about the paradoxical nature of this problem (and this student is considered to be a low achiever in mathematics!). I was particularly pleased when, a few months later, she remembered that lesson, and especially her moment of insight. However, I probably look back on it now with a more jaundiced eye and wonder whether I was sufficiently responsible to the mathematics of the problem. I think I would probably do the same thing but maybe in a follow-up lesson, use a 'what-if' approach – what would happen if we named fractions using Kate's way?; are Kate's and Jack's ways the same or different?; is there a relationship between them?; are there other ways that we might name the landing point? Indeed the conversation around these questions might lead to consideration of the history of fractions (especially Egyptian fractions) and would open up a myriad of opportunities for integration.

What I am saying - and I borrow this from a teacher who was implementing a problem-solving approach in her class - is that part of what we do as teachers is "becoming comfortable with being uncomfortable" (Mau, 2007, p.349). Much has been written about the different levels of discomfort that teachers can feel in that kind of situation (e.g., Frykholm, 2004) but the problem is that often we keep that discomfort private – it is not something that we bring out for general discussion with our teaching colleagues. But if we do let go and give children a voice, we can trigger these magical transformative moments in students' lives and indeed in our own lives as teachers. I know Sean (Delaney) is going to speak about building mathematical knowledge for teaching tomorrow and therefore my comments on the matter will be brief. Corcoran (2012)

[35] Misspellings have been corrected.

suggests that ‘contingency scenarios’ (Rowland, Huckstep and Thwaites, 2005) - the times when things don’t go as planned – are great sites for building mathematical knowledge for teaching. Why not bring that moment (of discomfort) to staff meetings and think about ways to deal with uncertainty? Thus there is an opportunity for you as teachers to think about your own mathematics and the ways that children learn and make sense of mathematics.

What are the implications for teaching? I suggest the need to go back to basics if we are to bring about any change and discuss what it really means to do mathematics. Are we concerned with a well-ordered clean subject or with messy, busy work? I think part of these discussions need to address the complexity of what it is we do as teachers. It is certainly not the case that the problem with mathematics teaching and learning can be addressed by bringing in the top graduates in maths and science into the classrooms to teacher (something I saw mooted in a recent newspaper article!).

We need to develop rich mathematical tasks and this means little more than making the ordinary extraordinary. So instead of asking children to consider the solution to “3 + 2” we might ask what two numbers add together to give 5. Some children might decide on 3 “and something”, others might proffer “3+2”, yet others might question whether 2+3 is the same as 3+2, or indeed whether zero is a number. What about fractions? Negative numbers? How many ways are there to add two numbers together to give 5? And, importantly, when and why should two numbers be added to give 5? Such an approach opens mathematics to all the children in the classroom. It is not a subject confined to those who arrive at solutions in a speedy fashion. Rather all children – whether they have exceptional talent or need extra support – can feel that they have a part to play.

Of course, there is the age-old problem of texts and tests. There is widespread acknowledgement among educationalists of the limitations of standardised tests and the issues surrounding “teaching to tests”. What is interesting to note is that when students use a problem-solving approach in their learning of mathematics, their scores on achievement tests improve (e.g., Boaler, 2009). It is good to see that many school staffs and individual teachers are considering less reliance on textbooks. However, their replacement must be something that engenders and fosters real mathematical thinking and cannot be just “more of the same.” An alternative approach might be to ask children to critique textbooks – for example, to select, from a page, problems, calculations etc. that are “relevant”, “interesting”, “easy”, “difficult” and so on. What I am saying is that I think there are lots of ways in which textbooks might be used differently with the consequence that there is a shift in the relationship between textbooks and the teacher and indeed between textbooks and the child. Where does teacher efficacy come from if children are not doing pages of textbooks or completing tasks very quickly? It is fostered in developing rich tasks, in predicting possible student reasoning as they engage with the problem, in generating and directing discussions and most importantly in those magic moments when children are brought in to mathematics (Smith, 1996).

I like this picture entitled “Counting in their Heads” painted by the Russian artist, Nikolay Bogdanov-Belsky in 1895. We see here children engaged in a very challenging mathematical task. Note the role of the teacher and the space that he gives to children for possibility thinking. We can almost hear the buzz in the classroom as they talk to each other, wander around thinking through the problem or as they talk, perhaps about their estimates, to the teacher. There are no concrete or digital materials to be seen and, yet, there seems to be mathematical activity in abundance. I am not for a moment saying that we should not use digital tools – these are the tools of today – but they must be used in a way that encourages children to think deeply about mathematics.

Finally, I return to the lesson on the grasshopper and to one pupil, Maeve, who at the close of the discussion said, ‘It is just like going to Cork – he will get there’.
Thank you.

References

Barton, B. (2013, March). *Why maths matters*. Paper presented at the HEA conference, University of Limerick.
Boaler, J. (2003). Studying and capturing the complexity of practice - the case of the 'dance of agency'. In N. A. Pateman, B. J. Dougherty & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 3 - 16). Honolulu: PME.
Boaler, J. (2009). *Equity and high achievement: The case of Railside School*. In D. Corcoran, T. Dooley, S. Close and R. Ward (Eds.), Proceedings of the Third National Conference on Research in Mathematics Education (pp. 2 - 19). Dublin: MEI.
Boaler, J., and Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171 - 200). Westport, CT: Ablex.
Cawley, M. (2011). *An investigation of affective variables in mathematics education: A case study in a primary school* (Unpublished Master’s thesis). Dublin City University, Dublin.
Corcoran, D. (2012). Constructivism made visible in contingency: Learning to teach mathematics in a community of practice. In F. Waldron, J. Smith, M. Fitzpatrick & T. Dooley (Eds.), *Initial teacher education: Perspectives on transformation* (pp. 135 - 154). Dublin: Liffey Press.
Dooley, T. (2010). *The construction of mathematical insight by pupils in whole-class conversation* (Unpublished doctoral dissertation). University of Cambridge, Cambridge.
Ernest, P. (2004). Postmodernism and the subject of mathematics. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 15 - 33). Greenwich, CT: Information Age.
Frykholm, J. (2004). Teachers' tolerance for discomfort: Implications for curricular reform in mathematics. *Journal of Curriculum and Supervision, 19*(2), 125 - 149.
Hersh, R. (1997). *What is mathematics, really?* Oxford, UK: Oxford University Press.
Mau, S. T. (2007). Under the microscope: Looking closely at one's own teaching and learning. In W. G. Martin, M. E. Strutchens & P. C. Elliott (Eds.), *The learning of mathematics: Sixty-ninth yearbook* (pp. 345 - 355). Reston, VA: NCTM.
Mendick, H. (2006). *Masculinities in mathematics*. Berkshire, UK: Open University Press.
O'Connor, J. J., and Robertson, E. F. (1999). *Zeno of Elea*. Retrieved from http://www-history.mcs.st-and.ac.uk/Biographies/Zeno_of_Elea.html
Perkins, R., Shiel, G., Cosgrove, J., Merriman, B., & Moran, G. (2013). *Learning for life: The achievements of 15-year olds in Ireland on mathematics, reading literacy and science in PISA 2012*. Dublin: Educational Research Centre.
Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*, 255 - 281.
Smith III, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*(4), 387 - 402.
Stern, E., & Mevarech, Z. R. (1996). Children's understanding of successive divisions in different contexts. *Journal of Experimental Child Psychology, 61*, 153 - 172.

Developing Numeracy for All Children: Supporting Teachers Supporting Learning

Seán Delaney, PhD, Marino Institute of Education

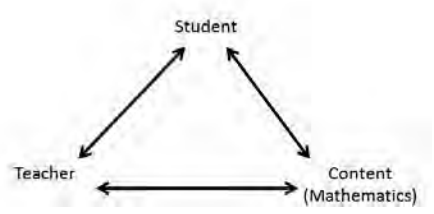
Good afternoon everybody. There's so much that I'd like to say to you about teaching maths, that it's difficult to narrow it down to a little over half an hour. I know that you are bombarded on a regular basis with ideas about how maths, and everything else, should be taught. We know that everyone has opinions about teaching. You have your own views that come from years of experience first as a student in school, then as a teacher, as a colleague of other teachers, and possibly as a parent or an aunt or uncle. You've also encountered eager salespeople who will try to convince you that the materials or the package they're selling will revolutionise how maths is taught and learned in your classroom. You read newspaper articles that either bemoan our national standard of achievement in maths or else report on some new development that promises great results if you adopt a particular approach.

So where do you turn to find helpful ideas about alternative ways to teach mathematics? The obvious answer would be towards sound educational research. But research on education is notoriously difficult because, unlike a randomised control trial as happens with new drugs, say, if you try one approach in teaching, it is difficult to replicate it exactly in several settings and very difficult to compare it to how another approach would have worked with the same children. Furthermore, research findings can sometimes appear to be contradictory. Add to that the fact that much educational research is written in language that is alien to practitioners. Although hopefully the *Irish Teachers' Journal* will change that.

In the next thirty minutes or so I won't be offering any quick fixes, or recommending any packages that will guarantee "amazing results or your money back!" Instead, I'm going to share with you some ideas and questions that have helped me in thinking about the teaching of maths. They include ideas about linkage in mathematics, teacher knowledge, Piaget's theory of knowledge, differentiated instruction, how you use textbooks, and a discussion about why changing teaching is difficult. What the ideas have in common is a belief that it's not the external resources you use, or the scheme you adopt that will change how maths is taught and learned in your school; it is only what teachers *do* in the classroom that will have any real, sustained impact on practice.

I'm going to put what is called the "instructional triangle" at the centre of what I'll say to you today. I'm using this image for two reasons. First, it offers a way to organise the ideas I want to share with you. A secondary reason I'm using it is because it reminds us that teaching is shaped by many factors and if you're going to change one factor you need to consider how it will interact with other factors. I'll say some more about this later.

The instructional triangle offers a way of thinking about the complex work of teaching in a way that helps us talk about it and understand it a bit better. At its simplest, the instructional triangle consists of three corners, representing the teacher, the child and the subject matter, mathematics in this case.



The Instructional Triangle (Cohen, Raudenbush, & Ball, 2003)

I know the word "instruction" may put off some of you because many educators in Ireland don't like it; we associate it with didacticism. But when I use the term I am using it to simply refer to "interactions among teachers and students around content" (Cohen, Raudenbush, & Ball, 2003, p. 122). It is the interactions among these three vertices that determine how good the instruction is: the teaching that leads to learning.

The first vertex of the instructional triangle that I'll direct your attention to relates to mathematics and the nature of mathematics. Many of us think about the topics in the maths curriculum in a discrete way. The contents page of a typical textbook will list topics such as place value, operations, multiplication, division, fractions and so on. However, most number and measures work in senior classes can be grouped into three themes: Multiplicative - as in multiplication - multiplicative reasoning, equivalence and computational fluency (National Council of Teachers of Mathematics, 2000).

Let me give an example. If children know their multiplication number facts, this should help them figure out their division number facts on the basis that if 7 x 8 = 56, then 56 ÷ 7 = 8. If you want children to practise division repeatedly, you could link this to testing if certain numbers are prime or not. This would give the children a reason to be interested in their answers.

Very often expressing one number as a fraction of another, or converting fractions to decimals are taught as discrete topics. Instead, where there is a remainder in a division problem, children could be asked to express the remainder as a fraction of the divisor. Or when children see that the answer to a division problem on a calculator differs from the answer they get using pencil and paper, they could be asked to convert the remainder to a decimal.

For the topic of equivalence, children could see that numbers can be expressed in different forms but that the value remains the same. Equivalent fractions are widely taught. But the term equivalent is used less often in relation to quantities in place value, hours and minutes, metres and centimetres, millilitres and litres and so on. By linking such topics, mathematical ideas are connected to and they reinforce each other.

The next part of the instructional triangle I want to look at is the teacher. In last Saturday's *Irish Times,* in an article about mathematics (Holden, 2013) the following comment was attributed to Seán Rowland, who founded Hibernia College:

> Maths is incredibly important: future employment is based on our knowledge of maths. What if we recruited the best engineers and mathematicians from other countries to teach in Irish schools for three to five years to get the standards up? Or we could employ maths graduates for a period in our schools after they finish college.

This idea has been mentioned from time to time both in Ireland and abroad. But the idea is flawed. It gives the impression that Irish teachers are not up to the job of teaching maths. That is clearly untrue. My study of the mathematical knowledge of over 500 teachers all around the country has shown that many Irish teachers have strong mathematical knowledge. My research also shows that the level of knowledge varies substantially among teachers; that shouldn't surprise any of us given that until last year it wasn't a requirement for prospective teachers to study any maths after their Leaving Certificate. And it's difficult to find a course where a teacher can develop their own mathematical knowledge, as distinct from learning about new ways of teaching maths.

But more fundamentally the stance is problematic because it gives the impression that in order to teach mathematics you just need to be good at maths yourself. This ignores the importance of knowing teaching methods for maths. But it also ignores the fact that the mathematics needed for teaching differs to the mathematics that is used by engineers and mathematicians. Engineers and mathematicians constantly try to compress their mathematical knowledge into more concise equations, like vacuum packing it, whereas teachers must try to decompress their knowledge of mathematics in order to make it more accessible to someone who is encountering some complex ideas for the first time. Let me give you two examples of the kinds of things teachers need to do with their mathematical knowledge.

When Deborah Ball addresses teachers, she often asks them to calculate the answer to 35 x 25. Most people will answer 875.Okay. But supposing you're correcting children's work at home and you come across this answer.

$$\begin{array}{r} 35 \\ \times\ 25 \\ \hline 255 \\ +\,800 \\ \hline 1055 \end{array}$$

Can you use your mathematical knowledge to figure out how a child might have got this answer?

The student probably said "5 x 5 is 25, put down the 5 and carry the 2. 2 plus 3 is 5, five 5s are 25. Put down the zero. Then Two 5s are ten, put down the zero and carry the 1. 1 plus 3 is four, two 4s are eight." This is the kind of problem that teachers need to solve every day. The mathematical knowledge you need to solve it is not the kind of maths that engineers or mathematicians use in their work. Here is another problem where the students got different wrong answers. The question here is to figure out which students are making the same kind of error.

13. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I) $\begin{array}{r} {}^{1}\ \ \\ 38 \\ 49 \\ +\,65 \\ \hline 142 \end{array}$ II) $\begin{array}{r} {}^{1}\ \ \\ 45 \\ 37 \\ +\,29 \\ \hline 101 \end{array}$ III) $\begin{array}{r} {}^{1}\ \ \\ 32 \\ 14 \\ +\,19 \\ \hline 64 \end{array}$

Which have the same kind of error? (Mark ONE answer.)

(from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf).

The answer is I & II - the children over-generalise the "carry 1" rule. In some research I did in the United States, mathematicians knew that the answers were incorrect but many found it difficult to state how the children had gone wrong (reported in Hill, Dean, & Goffney, 2007). That's why it is teachers and not engineers or mathematicians who are best placed to teach mathematics to children.

Liping Ma compares a teacher's knowledge of mathematics to a good taxi driver's knowledge of a big city (Ma, 1999). Most residents know certain parts of a city well - near where they live or used to live, along their commuter routes, and on other routes along which they frequently travel. But a taxi driver's understanding of a city is different. Taxi drivers know the best route to take at different times of day; they know which sites to show a visitor who is in the city for the first time and no matter where they pick up the fare they can get to where they need to go without using a sat nav. A teacher needs to know mathematics in a similar way - beginning from where the children currently are, knowing where they need to go, what possible routes to take, and knowing which topics are connected to one another and how.

The side of the triangle that links the children and the mathematics refers to how children learn mathematics. Here I like the distinction Constance Kamii makes (Kamii, 2006), drawing on Piaget's work, among three different types of knowledge. Although the distinctions are quite general and probably over-simplify the idea, they offer a useful way to think about how teachers help children engage with mathematics. Kamii distinguishes among physical knowledge, social-conventional knowledge and logico mathematical knowledge according to the ultimate source of the knowledge.

Physical knowledge is something that you can experience through your senses. A pencil is made of wood, sponge is soft, rock is hard. Social conventional knowledge relates to conventions that are used in a society - we drive on the left side of the road, how we set the table, the language we speak, and whether we use metric or imperial measures - centimetres or inches. These are things we learn through observation or being told what to do. The third type of knowledge is logico-mathematical knowledge. This consists of relationships we make among things, such as similar, different, the number five. We cannot see the number five; it's an abstract idea. We can see several examples and representations of five and then make the conclusion that what the representations of five have in common is that they have the same quantity or the same position.

Likewise with similarity and difference, you and I can both look at a set of pencils and say that they are the same - in length, sharpness, materials, or different - in colour. These are ideas we learn by connecting objects and ideas in our minds. Observation alone is not enough. Logico mathematical knowledge refers to the kind of abstract knowledge that maths is. According to Kamii, the problem with learning mathematics is that sometimes maths is treated as if it is physical knowledge (which is a risk of relying too much on manipulatives) or as if it is social-conventional knowledge (that someone else can tell it to you and you can just listen). Instead, if children are to learn mathematics, their *thinking* must be stimulated, and children will learn maths in different ways and at different times as they make internal connections between objects and patterns that they experience. Many of us learned mathematics by memorising parts of it. Indeed some memorisation is essential - for definitions and number facts for example. The problem with memorisation is that if it doesn't make sense to children, they stop thinking and try to rely on memorisation alone. This makes it much more difficult for them to learn mathematics.

Returning to the instructional triangle, we know that in any class children will differ as a result of many factors. First of all, children differ from one another because many of them are taught in multi-grade settings. Differences can also be due to prior opportunities to learn maths, or to social, economic, psychological, or linguistic factors. I prefer to refer to children as higher achieving or lower achieving rather than using terms such as weak, average, bright, strong at maths and so on. Those terms tend to lock children in boxes and place teachers in a passive role in terms of how they can impact on children's learning. Those terms also lack the kind of nuance that captures a child who does well on tasks requiring knowledge of shape and space and who can reason logically but has difficulty, say, with number topics or in communicating mathematical ideas.

Faced with such differences among children that exist in most classes, teachers need to differentiate their teaching. This is represented in this line connecting the teacher and the children on the instructional triangle. One way to differentiate is to use problems that can be approached at different levels by different children but all children can be involved in discussing them. Here children are asked how a zoo keeper can create an enclosure for sheep that is 100 square metres in area. This example is taken from a replacement unit on the topic of area for fifth class that I worked on with the PDST. A replacement unit presents a mathematical topic that can be used alongside your usual textbook but instead of doing the chapter from the book, you use the replacement unit. Here is one problem from it:

Task 10

Solve this problem with a partner.

Dublin Zoo has just received two new sheep for the Family Farm part of the zoo. The zoo keeper wants to build an enclosure for the sheep. She decides that the enclosure must be square or rectangular with an area of exactly 100 square metres.

(i) Which different enclosures could she build?

(ii) How many metres of fencing will she need for each possible enclosure?

(iii) Use your copy or some graph paper to draw all the possible rectangular or square enclosures.

(iv) Include a key to tell how much each unit on your copy or graph paper equals.

(v) Which enclosure would you recommend that the zoo keeper builds? Why?

(From: http://www.pdst.ie/sites/default/files/PDSTArea5Maths%20Pupils%20.pdf)

First the problem encourages the children to work with a partner. When I gave this problem to fifth class children, some did well to come up with one configuration whereas others came up with configurations that involved fractions (12.5 m x 8 m). But all the children can discuss the answers together. This helps all of them to be involved in the discussion after they have worked on such a problem.

Now I'd like to share with you a transcript of some children discussing their answers to this problem. The background to the exchange is that it occurred in a maths laboratory setting, where what is offered is not model teaching but public teaching - teaching that can be discussed by educators. The class consists of 20 children in fifth class, recruited from different schools, four of them DEIS, mostly around Marino. Twenty educators were observing the class. The children knew what perimeter and area were. The problem had already been discussed so that all children knew what they had to find out. They were not told how to solve the problem. The transcript records a small part of the children's discussion of their solutions to the problem.

SD:	Okay thanks F________. Okay. Thank you girls that was a very nice presentation and you were able to answer the questions that you were asked as well thank you. Do you want to go next? Okay, we'll let K_________ and J_________ go next.
J:	Well, we think the twenty five metre and four metre across because it's a rectangle and we just think it's a good shape.
K	And we also did fifty and two, two across and fifty down.
SD:	Okay, so you did one that was fifty metres long and two, two metres wide. This one here. Okay so it's fifty metres long and two metres wide, that's this

	one here, okay.
K:	And the other twenty five metres down and four metres across
SD:	Fours metres across and you're recommending which one again?
K/J:	The two.
J:	But I think the fifty is too skinny.
SD:	It's too skinny. Okay and do you agree with that K_________?
K:	Yeah, but I still like the two of them.
SD:	Okay, you like both of them. You didn't have any one that the girl... that M________, N_________ and A___________ had you hadn't any other one. They mentioned all the other ones you had?
K:	Oh we also had ten on by ten, multiplied by ten
SD:	I think you had that as well. Ten metres long and ten metres wide. Okay that's fine thank you very much K_________ and J_______.
D:	Seán, I have a question for them.
SD:	D______ has a question for you.
D:	Why do you do those two because the perimeter of those is quite bigger than a few of the other ones, and they could be more expensive to buy fencing for them?
K/J:	I don't understand what you're saying.
SD:	Can you explain yourself a little bit more clearly D______?
D:	Alright, well the perimeter of those two are bigger
SD:	Of which two now?
D:	Of the two type of fields that they're ...
SD:	That they're recommending
D:	That they're recommending
D:	Three of the other ones have smaller perimeters and it would be cheaper to buy fencing for those three apart from your two which are very, quite expensive.
SD:	Do you understand what D_______ is saying?
K/J:	Yeah.
SD:	So, what is he saying?

J:	The other three that we did that we didn't recommend why didn't we pick them?
SD:	Why does he think you might have chosen another one rather than the one you did?
K/J:	Because it would be more expensive
SD:	What would be more expensive?
K/J:	The fencing.
SD:	For which configuration?
K/J:	For the ones that we did recommend.
SD:	Is that what you're saying?
D:	Yeah, the ones that they were saying were...
SD:	Let's just take one of them. I think the one you both agreed on was twenty five metres by four metres isn't that right? So, what's your point D______? Can I ask you all just to listen to this conversation? Maybe just leave down the water and things like that for the moment pens and pencils and just concentrate, 'cause I know it's important that you concentrate on this.
D:	That is the longer perimeter
SD:	Twenty five multiplied by four.
D:	Yeah. Twenty five multiplied by four say, say that twelve point five multiplied by eight, twenty by five or ten by five and that means that it will be more expensive to buy fencing for that one than those three. So, why did you pick that one?
K:	Amm, I picked that one...
SD:	Sorry, can I ask D_____ can you actually just put into your own words what D_____just said?
D:	He said; why did you pick them two 'cause they would be a lot more expensive if you didn't pick them like if you picked the other one it would be a lot cheaper.
SD:	And why would it be more expensive?
D:	Because like there's more met...square metres
SD:	More square metres?
D:	Yeah, like more, like the perimeter is larger than the other ones. Like say they picked, they went for twenty five by four, so the perimeter of that like would be about fifty eight, the other one ten by ten that's only forty

SD: So do you agree with D______'s point, do you want to then respond to it K_______?

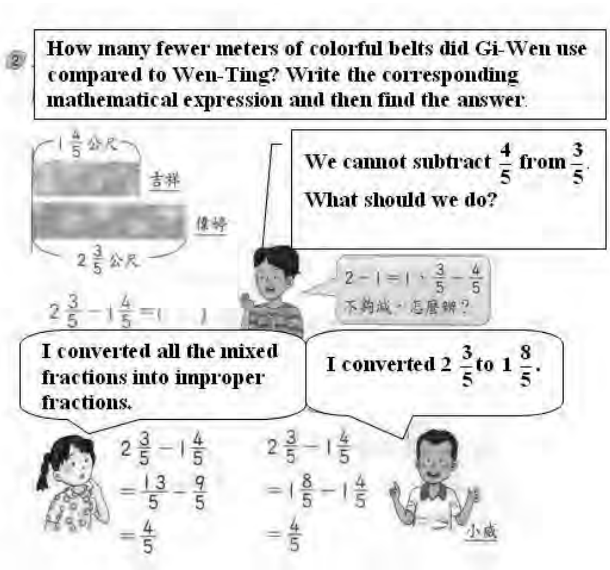
K: We picked the twenty five by four because the other one, the fifty by two we felt was too skinny and if the sheep were to walk around they'll be up real tight in it. The other two that we crossed out that we didn't want to do, well it was really long but really skinny and the other one didn't work because it was twenty five, twenty five.

SD: So, would I be right in saying that when you considered recommending, you didn't really take the cost of the fencing into account? Would that be fair enough? You were kind of thinking of how comfortable the animals would be and if you went for the long really skinny one, that they'd...

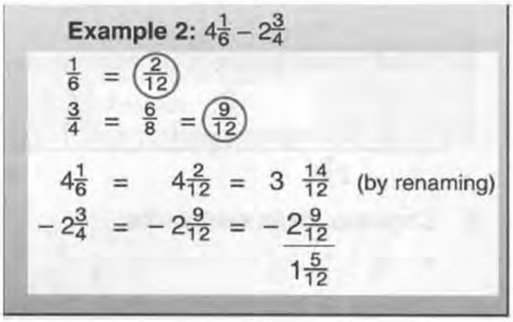
J/K/SD: They would be squished together.

I'd like you to take two things from the transcript. The first is that children can learn mathematics by working on a problem - they don't need to be told what to do in advance. Second, revoicing - where children are asked to repeat or paraphrase what another child said - can be used as a strategy for differentiating.

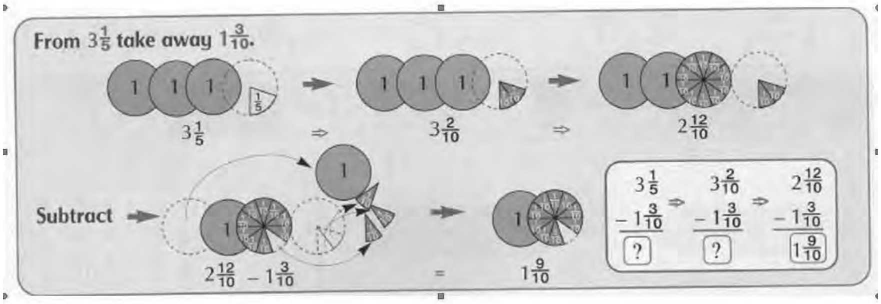
An important resource for teachers in teaching mathematics is the textbook. Although many people believe that teachers should reduce their use of textbooks, I am not one of them. I believe that teachers need to focus their energies on teaching and what it entails rather than having to design resources for use in class - in 11 subject areas. Textbooks are important and useful as an interpretation of what is in the curriculum. They can help to structure your work for the year. However, I have a difficulty with the quality of mathematics textbooks that are currently available to us in Irish schools. I can best illustrate this with some comparisons of how content is covered in Irish textbooks and textbooks in other countries. These relate to the topic of addition and subtraction of fractions in fifth class. Here is an illustration of worked examples from a textbooks in Taiwan (Mou & Yang, 2005).



Here are some worked examples from similar textbooks in Ireland:



(Courtney, 2002)



(Barry, Manning, O'Neill, & Roche, 2003)

Several differences are clear. The Taiwanese example addresses the children and asks some questions as they work through the example. It offers more than one way of doing the problem and it grounds the example in a believable situation.

There are more general problems with textbooks. One is that the content becomes prescriptive so that teachers feel they have to get through as much of the book as possible rather than taking time to discuss mathematical ideas with children and helping them to communicate mathematically. Another problem is that with the exception of the revision chapters, the questions in most units typically relate to the topic of the chapter. So at the end of the multiplication chapter, all the questions require you to multiply, at the end of the division chapter they require you to divide and so on. Consequently, when children have to look at the problems in a test and decide which operation to use, they can be lost because they haven't been used to deciding which operation is needed.

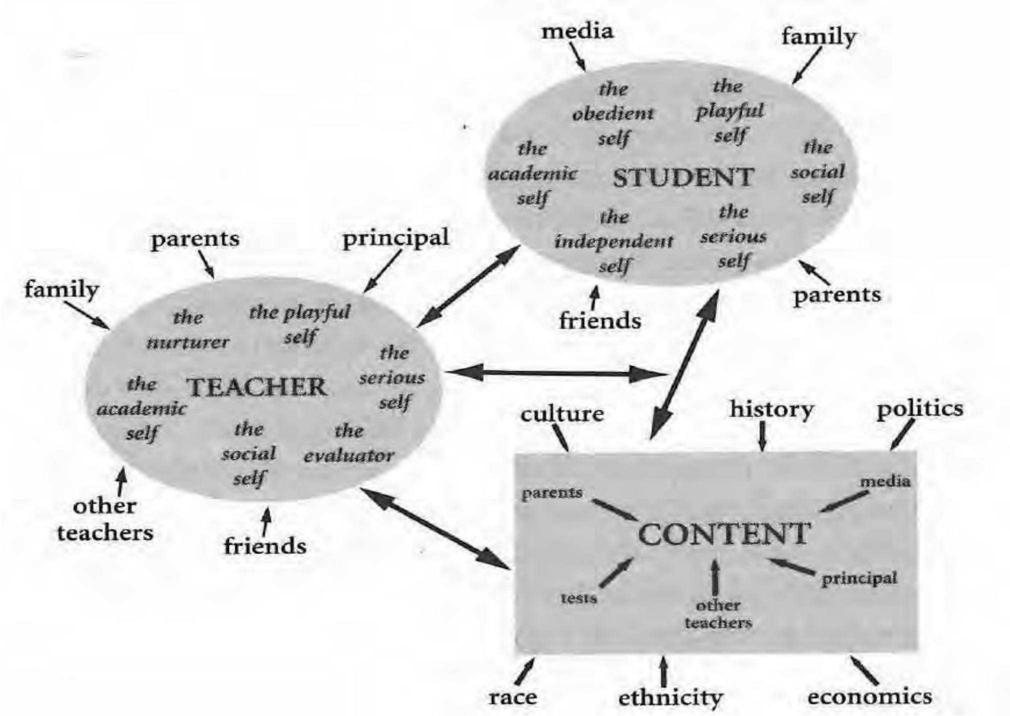
Why are the textbooks so bad? Textbook publishers say that they give teachers what the teachers want. I'm not so sure that's the case. Do the publishers really consult with you to find out what you want? What kind of alternatives do they offer you? It would be good to see textbook authors drawing on ideas from textbooks in other countries as well as on local ideas and indeed mathematicians may well have a role alongside teachers and mathematics educators in this work. In the replacement unit on area, which can be downloaded from the mathematics section of the PDST website, we tried to lay out what we would like to see in a textbook for fifth class.

Because textbooks are unlikely to change soon, you can try to compensate for their shortcomings by asking children questions that will help encourage them to think mathematically. Such questions include the following:

What do you think about...?	How did you get that?	Estimate
Are you saying that...?	Put in your own words...	Why?
Put into your own words	Tell me about....	Explain

Could it be done another way?	Is there a pattern?	Compare
How does that connect with...?	Put into your own words	Describe
Do you agree or disagree?	How can you tell?	How can you be sure?

I'd now like to stand back and look at the instructional triangle from a different perspective. I mentioned before that the model simplifies teaching. One researcher who has given some thought to this is Magdalene Lampert (2001). She shows that the instructional triangle that I've shown you is much more complicated given the various factors that influence what happens in instruction.



(Lampert, 2001)

According to her model, no aspect of the triangle is stationery and no part of it is wholly independent. Teachers will play several roles - at times nurturing and at times evaluating for example. The model shows, too, that parents have an influence on what teachers and students do in class. If you try and make a radical change in your teaching approach and don't consult with or inform parents, they may be concerned about what you're doing. It's probably no coincidence that the current emphasis on literacy and numeracy is coming at a time of recession, an economic influence. Various factors are at play that influence whether or how change can happen. Returning to an earlier point, you can have a great textbook that is associated with not-so-great teaching and you can have great teaching associated with a not-so-great textbook because the teacher and the textbook interact with each other - neither has a passive role in instruction. If teachers are aware of the various influences on them, on students and on the mathematics content, they will understand better what change is possible and how it might come about.

Another feature that Lampert identifies as having a possible impact on mathematics teaching is culture. Mathematics lessons in Ireland are quite predictable. Typically a lesson has four components. First, we check homework or engage in a warm-up activity such as checking tables. Then we demonstrate how to solve problems for the day by presenting a few sample problems and doing them or asking children to do them on the board. Then we ask the children to practise problems similar to those for which solutions were demonstrated and finally we correct the class work and assign homework. This is the dominant pattern identified by Stigler and Hiebert (1999) in lessons in the United States and one with which many Irish teachers I've spoken to and observed identify.

When you think about it, it's not surprising that most of us teach in a similar way. That is because teaching is largely a cultural activity (Stigler and Hiebert, 1999). To explain how a cultural activity works I need to move away from teaching maths for a minute and tell you about a celebration in another country.

A couple of years ago a friend of mine was in Malaysia for their national day, Hari Merdeka. At the centre of the celebration is a parade with floats, a flypast by the Royal Malaysian Air Force, flag raising, patriotic songs, dancing and fireworks. People dress up in wigs and masks, they sing and blow on vuvuzuelas. But what amazed her most was what happened after the various events. People just hung around the streets talking and went home peacefully. No alcohol was consumed.

Think how different that is to our national day of celebration on St. Patrick's Day. We'd have the parades, the songs, the dancing and the fireworks but interspersed throughout the whole day is drinking. Later that night hospital emergency wards are on alert because they know patients will be admitted because of over-consumption. This is because our attitudes to drink are shaped by our culture. Children learn this from an early age. When a young child stumbles, an adult will jokingly say "you must be drunk" and underage drinking is accepted as a "fact of life", and prominence is given to visitors - most recently Barack Obama or Queen Elizabeth - sipping alcoholic drinks. Experiencing such events and encountering such attitudes help us learn how to behave in a culture.

That is also how we learn about teaching. It is something that is learned through observation and participation in schools, first as students and later on teaching practice and then working in schools. When we were students in school we observed teachers all day every school day for at least 13 years. The impact of a few years in a college of education afterwards can be pretty weak by comparison.

So, if you want to change how maths is taught in the country as a whole, it is like putting in place a plan to celebrate St. Patrick's Day next year without alcohol being consumed in the country. Bringing about such a change seems like a daunting challenge. It could be done, as with the smoking ban in the workplace, but it would take some even more courageous policy decision for it to happen. Making fundamental changes to teaching requires similar, enormous effort on the part of educators, policymakers and the wider community. In the absence of such an initiative, it is the changes that you can bring about in your classes and in your schools that will change mathematics teaching. This can be done by linking topics together in new ways, by being more conscious of the mathematical knowledge you use when teaching maths, by helping children learn mathematics by thinking about ideas as well as by memorising them, by using strategies that help you differentiate learning for your pupils, by compensating for shortcomings in our textbooks and asking publishers for changes in future editions, by anticipating how any change you make may have knock-on effects on other parts of the system, and finally, by being somewhat counter-cultural.

References

Barry, K., Manning, P., O'Neill, S., and Roche, T. (2003). *Mathemagic 5*. Dublin, Ireland: CJ Fallon.

Cohen, D. K., Raudenbush, S. W., and Ball, D. L. (2003). Resources, instruction and research. *Educational Evaluation and Policy Analysis, 25, No. 2*, 119-142.

Courtney, D. (2002). *Action Maths*. Dublin, Ireland: Folens.

Hill, H. C., Dean, C., and Goffney, I. M. (2007). Assessing elemental and structural validity: Data from teachers, non-teachers, and mathematicians. *Measurement: Interdisciplinary Research and Perspectives, 5*(2), 81-92.

Holden, L. (2013, November 9 2013). Why are the Irish so bad at maths - and how can we solve the conundrum?, *The Irish Times,* p. 2.

Kamii, C. (2006). Measurement of length: How can we teach it better? *Teaching Children Mathematics, 13*(3), 154-158.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, Connecticut: Yale University Press.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, New Jersey: Lawrence Erlbaum Associates Inc.

Mou, L., and Yang, J. (2005). *Mathematics 4 (Vol 2)*. Taipei, Taiwan: Kang Hsuan.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Stigler, J. W., and Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

7

Discussion Groups and Workshops

Introduction

Delegates were assigned to different discussion groups to facilitate closer examination of some of the issues that arose from the conference documentation and presentations. Each one of the six discussion groups was given a list of questions to guide discussion. Members of the INTO Education Committee acted as facilitators and rapporteurs. The collated responses of the participants are outlined below.

Literacy and Numeracy Strategy

Most participants reported that more time is being spent on maths, in line with the requirements of the Strategy, although some reported that they were already spending extra discretionary time on the teaching of maths prior to the introduction of the Strategy. Schools, mainly through the School Self Evaluation (SSE) process, have reflected on their teaching of maths, and methodology changes have been taking place. These include more practical approaches, and the extension of the use of concrete materials in all classes. Station teaching, group work, and to a lesser extent, team teaching were all mentioned, although constraints, in the form of resources and of time were put in context. Class size and staffing levels arose as issues. Changes in planning were noted, with many teachers now placing emphasis on agreeing maths language on a whole school, co-ordinated basis. Whole school initiatives, such as Maths Week, are beginning to take place in schools. The negative impact of increased paperwork was cited as a concern. In brief, the Literacy and Numeracy Strategy appears to have given teachers an opportunity to focus more reflectively and more analytically on teaching and learning in maths.

Challenges in the classroom

Teachers cited class size and multi-grade teaching as major challenges in their classrooms. The attitude of children to maths, often caught from negative attitudes expressed by their parents, is presenting as a major obstacle. An overemphasis on number, to the detriment of other strands, and a sense of an overloaded curriculum is also challenging. Lack of adequate resources, and of storage and display space in crowded classrooms, is an underlying issue. The integration of children with learning difficulties, and with language difficulties, is proving problematic, as is the relative lack of learning support for maths in general terms. The integration of learning support teachers into classrooms for team teaching purposes was mentioned by some teachers as beneficial.

Problem-Solving

The teaching of problem-solving has been influenced by an inherent over-reliance on textbooks in the Irish classroom and a gap in the content and approaches used in textbooks. Problem-solving is often just tacked on to a lesson, and in some instances, left to the end of the year. Textbooks are also perceived as focusing on word problems, rather than problem-solving itself. The children's restricted daily lives, and relative lack of opportunities to experience maths in real life settings, is impacting on their reasoning abilities. There is also a mismatch between the language used at home, and that used in maths class in school (examples cited as the use of imperial measures in the home and the use of metric terms in textbooks and school). There needs to be more emphasis on the hand on use of concrete materials, and on making problems child relevant, and congruent with the child's own experience. The recent maths initiatives in DEIS schools have had such a positive impact that teachers now feel they should be made available across the system, and their introduction funded and resourced. This would entail more CPD for teachers.

Language

There was a general agreement that mathematical symbols are being introduced too early in the child's school experience, with a perceived over reliance on textbooks being a factor. The compelling urge teachers have to complete textbooks, grounded on parental perceptions, needs to be addressed, to free up time to properly utilise concrete materials, and flesh out real life mathematical experiences. The use of concrete materials needs to be broadened out through all classes, and teachers need to be encouraged and supported to do this. Teachers need to be alert to the fact that not all children automatically understand mathematical language – it needs to be taught discretely.

Schools are now finding it essential to agree on the consistent use of language in maths, and that children must be proficient in the language of maths, prior to the introduction of symbols. Teachers were in agreement that discrete time and attention must be given in all maths lessons to the relevant maths language being used in order to lessen confusion on the part of the learner. Problem-solving needs to be relevant to the child's own experience, and based on the use of concrete materials.

Resources

Resources need to be varied, and there is a need to engage with differing learning platforms as the use of ICT can transform maths lessons. The use of a maths week initiative, or the Maths Eyes experience, have been greatly praised as being highly effective in 'de-textbooking' the maths experience. One group pointed out that we, as teachers need to 'rediscover it ourselves'.

Learning Support

Learning support-provision in schools has traditionally targeted literacy and language issues, with numeracy often being an add-on. The cut-backs in support for children who do not have English as a mother tongue have also impacted on learning support provision in general, and there is a perception that numeracy has been somewhat squeezed out. Provision of learning support in numeracy varies greatly across the whole school spectrum, and is individual to each school's context. The integration of learning support teachers into classrooms for team teaching purposes was mentioned by some teachers as beneficial.

Differentiation for high achievers

Teachers in general perceive the issue of differentiation as both a curricular and pedagogical challenge, for which they feel ill prepared from an initial teacher education perspective. Extra materials and resources need to be put in place if high achievers' talents are to be developed from an early stage. Differentiation for high achievers could be an area for targeted learning support, but given the resource challenges being met by schools in recent years, it will probably continue to be area of low prioritisation for schools struggling to provide adequate support for the weaker pupils.

To be effective teachers of maths:

Good communication is an essential part of any effective teachers' skill set, and this is especially so in the teaching of maths. It needs to be coupled with a love of the subject, and an ability to engage with guided discovery methodology – maths must be put daily into real life contexts for pupils. Teachers' own self confidence in the teaching of maths would benefit from the provision of adequate resources, and of on-going CPD opportunities.

CPD needs

The praise being expressed by teachers from DEIS schools has prompted a call out from teachers across the system to have the maths initiatives currently in use (Maths Recovery, Ready Set, Go Maths) available more widely. The potential for resources to be made available through websites was commented on, and teachers cite many websites regularly used by them in their classroom work. On-site whole staff in-service was mentioned as being effective, but teachers show a certain reluctance to engage in this during the summer break.

Initial qualification issues

Opinions varied as to the level of qualifications, from an insistence on a high mark in the Leaving Cert (at least an A2 at ordinary level), to those who sensibly make the suggestion that a teacher who had struggled themselves with numeracy issues, but overcame them through being effectively taught, would have a greater understanding of weaker students than someone to whom maths came naturally and easily. Any DES commitment to improving numeracy levels in schools should be backed up by the provision of adequate resources and focussed and relevant CPD.

Assessment

Teachers use a broad range of assessment approaches in maths, from teacher observation, to teacher designed tasks and tests, culminating in annual standardised tests. The Standardised Tests appear to be reported to parents twice annually – at the annual parent teacher meeting face to face, followed with a written end of year report. If there are any problems, testing and reporting occur more frequently, and are pupil specific. Difficulties with parental perception of the STEN score are arising, with parents wanting high scores for their children, and not being content with the average of 5 or so. Standardised tests are helpful in identifying pupils for learning support, and, when analysed, can feed into school planning by identifying areas that need attention. A worrying trend is being reported, of teachers feeling pressurised to have good test results, and there is emerging anecdotal evidence of teaching to the test taking place.

Parental Involvement

Teachers were of the view that parents need to know how maths is being taught to their child, and that this could be done by the use of a booklet outlining mathematical terms and approaches, or by information evenings. The potential for parents to help encourage and develop problem solving, through shopping, cooking, telling the time, the use of menus, timetables and brochures, and through an approach where maths is seen as being integrated into the everyday environment , is seen by teachers as vital in raising maths abilities and standards. However, parents must be enabled to do this, by their closer integration into their child's experience of maths.

Textbooks

While teachers are sensitive to the limitations of textbooks, they are somewhat reluctant to dispense with them entirely. They are seen as essential in a multi-grade setting, and can be a good guide to a topic, as well as a help in structured planning. Textbooks appear to have improved recently, with lesson plans and termly and yearly plans being offered in teachers' resource books. Some teachers feel older textbooks had better content. Teachers would broadly agree that textbooks are a valuable resource, as long as they are used properly. The parental perception that every page, and indeed every sum, needs to done for effective teaching to have taken place, is seen by teachers as unduly restrictive and often damaging. Teachers advocating a move away from textbooks completely will still admit that it requires a lot of time and resources to build up effective and useful teaching material.

Use of ICT

The variety and extent of resources now available on websites have made ICT an integral part of teacher preparation. Their use by pupils for classroom learning is somewhat constrained by limits on resources – it depends on how many computers/devices are available for pupils to work with. Their use in learning support appear more consistent than in classrooms in general, but this is an evolving field.

Planning

A variety of resources were cited as being useful, from the curriculum books, to textbooks and their accompanying teachers' books, to computer software available, all the way through to the NCCA planning tools and templates available from the PDST. Those involved in Maths Recovery and similar programmes also use them to inform their planning.

Tables

There was widespread and almost unanimous support for the continued use of tables, and for the teaching of long multiplication and long division, despite the use of calculators in secondary school. Some curriculum issues arose, with the long standing issue of an overloaded curriculum arising, as well as a perception that in some areas the curriculum is not challenging enough – infants being cited.

Transition

Transition issues of continuity and curriculum content in first year need to be addressed urgently, and the upcoming reform of the Junior Cycle is an opportunity for on-going concerns to be voiced. One group expressed a concern that feeder schools should meet locally with secondary schools to address issues and concern.

Workshops

In addition to participation in the discussion groups, all delegates got the opportunity to attend two workshops. The workshop presenters and their subject areas are listed below:

John O'Shea, Mary Immaculate College

Teaching mathematical problem solving in the primary school: changing behaviours

Liz Dunphy, St Patrick's College

Children thinking and talking about mathematics in infant classes

Noreen O'Loughlin, Mary Immaculate College

Maths Recovery: moving from intervention to innovation as way of effecting change in maths teaching and learning

Patsy Stafford, Froebel Department of Primary & Early Childhood Education (NUIM)

Children's understanding of place value: implications for teaching

Siún Nic Mhuirí, St Patrick's College

Maths talk: Talking about mathematical thinking

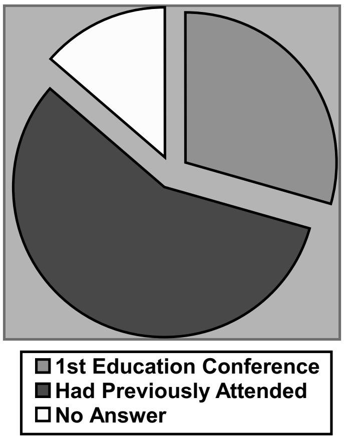
Tracy Curran, Cloneen National School, Tipperary

Keeping it real! Taking a fresh look at problem-solving in the primary school classroom

Appendix III – Conference Evaluation

Summary of Participant Evaluations

This year the INTO Consultative Conference on Education took place in the Heritage Hotel, Portlaoise on Friday 15 and Saturday 16 November 2013. There were 339 registered delegates, in addition to national committee members, presenters and guests. Evaluation forms were circulated to all delegates and a total of 145 were returned for analysis.



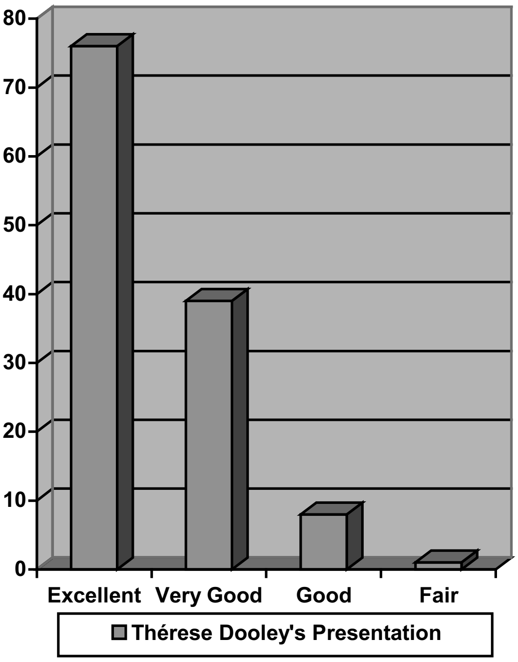
Of those delegates who indicated whether they had attended an Education Conference previously, 34% were first time attendees (a slight increase on the previous year) and 66% had previously attended. The Conference has a starting time of 3pm which aims to facilitate, in some way, those teachers who have to be in school that day. There is no substitute cover for delegates attending the Conference.

INTO President, Brendan O'Sullivan officially opened the Conference on Friday and outlined the timetable for the evening. Dympna Mulkerrins, Chairperson of the Education Committee, then took delegates through the work of the committee and how the discussion document prepared for the conference had been achieved. Deirbhile Nic Craith, Senior Official, outlined the national and international context for the current focus on numeracy in our education system. All speakers referred to the background document which had been circulated to all delegates prior to the conference. This document was rated as either 'excellent' or 'very good' by 80% of respondents.

Anne English, who represents District 8 on the Education Committee, gave a very well-received presentation focussing on numeracy in our primary schools, and some of the changes that have come about following the introduction of the National Strategy to improve Literacy and Numeracy. Anne's presentation was rated by 85% of delegates as either 'excellent' or 'very good'.

Dr Thérèse Dooley, St Patrick's College Drumcondra, then addressed delegates on the topic of 'Teaching Mathematics: The power of uncertainty'. This was rated very highly by delegates, over 90% of whom rated the presentation as 'excellent' or 'very good'.

Delegates then moved to their separate discussion groups to consider a number of questions on the topic of numeracy. These groups are always lively and generally feature in evaluation forms as one of the most popular aspects of the conference. This year there were some issues with the size and sound-proofing of some of the breakout rooms and this was reflected in comments on the evaluation forms. Despite these physical problems, the level of discussion and engagement was good, with one delegate noting 'I really enjoyed the discussion group – great to hear other opinions and hear what works / doesn't work in other schools'.



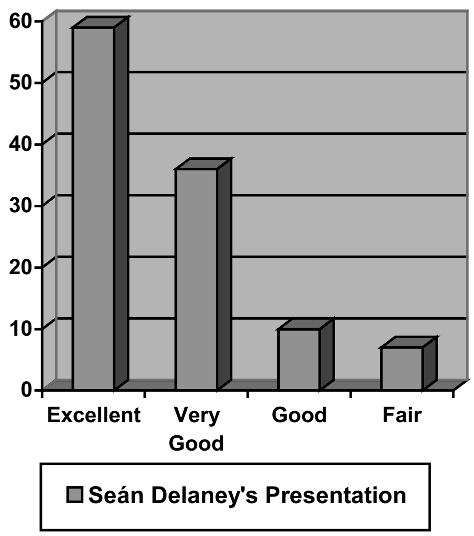
The Friday evening was brought to a close with the launch of the *Irish Teachers' Journal*. Professor John Coolahan, NUI Maynooth, gave a presentation to officially launch the first edition of the *Irish Teachers' Journal* and there was a drinks reception to celebrate the occasion.

On Saturday morning delegates went straight to the first of their two workshops. These were highly rated by delegates, however the limitation of the size of the rooms was noted again. The presenters were:

Presenter:	**Theme of Workshop:**
John O'Shea	*Teaching mathematical problem-solving in the primary school: Changing behaviours*
Patsy Stafford	*Children's understanding of Place Value: Implications for teaching*
Siún Nic Mhuirí	*Maths Talk*
Tracy Curran	*Keeping it Real! A fresh look at problem solving in the primary school classroom*
Liz Dunphy	*Children thinking and talking about mathematics in Infant Classes*
Noreen O'Loughlin	*Maths Recovery: Moving from intervention to innovation as a way of effecting change in maths teaching and learning*

As in previous years, due to the logistics of organizing workshops, delegates were assigned to workshops rather than given the opportunity to choose which one they would like to attend. This fact was noted in the comments by a number of delegates who felt they would have liked the opportunity to choose their workshop. Some delegates changed groups themselves to target the workshop they felt would be of particular interest to them or their school situation.

Notwithstanding this, the response to the workshop presentations was very positive, with 92% of delegates rating them as either 'excellent' or 'very good'.



Final Session

Following the workshops, delegates returned to the main hall for the final presentation, given by Dr Seán Delaney, Marino College of Education, on 'Developing numeracy for all children: Supporting teachers supporting learning'.
This keynote presentation was also very well received, with over 80% of delegates rating it as either 'excellent' or 'very good.

Conclusion
Overall, the feedback from delegates suggests that the conference was very successful. The theme of *Numeracy in the Primary School* was well-received and timely given the implementation of the National Strategy to improve Literacy and Numeracy.

For the last three years, the Education Conference has operated at maximum attendance. While it is easy to accommodate large numbers of delegates in a plenary set-up, it is more challenging to accommodate the 6-7 workshops that run concurrently. This year the hotel, which was chosen for its 'Fair Hotel' status, struggled with the numbers and this was reflected in the feedback from delegates. Issues with sound-proofing between discussion groups were resolved on the Saturday by moving one of the discussion groups to the main hall, but this did effect the experience for number of delegates. From a content point of view, the speakers were very well received, and the workshop presenters came in for particular praise.

When asked for suggestions to improve the conference, delegates were clear that the group work and the workshop-style presentations were what they were most in favour of. The only drawback noted was the inability of delegates to attend all the workshops – 'if workshops were slightly shorter, maybe we could attend more? I am disappointed I didn't get to partake in some'.

Overall, *Numeracy in the Primary School* and Education Conference 2013 was successful and well received by all those attending.

Sample of Comments from Delegates: 2013

- The whole experience was brilliant. I would have loved to have attended every workshop. Looking forward to the report. Thank you so much, my teaching will change on Monday!
- Education Conferences are always so relevant!
- Myself and my companion were assigned to different discussion groups, so we both attended different workshops, this was great as we can now co-ordinate our notes and present this to our branch staff.
- Launch of the Education Journal – it's great to see, it's about time there was such an outlet in our field.
- An ghné is fearr .... Plégrúpaí agus an caidreamh a bhí idir Múinteoirí.